Parametric Surfaces

CSCI 4229/5229
Computer Graphics
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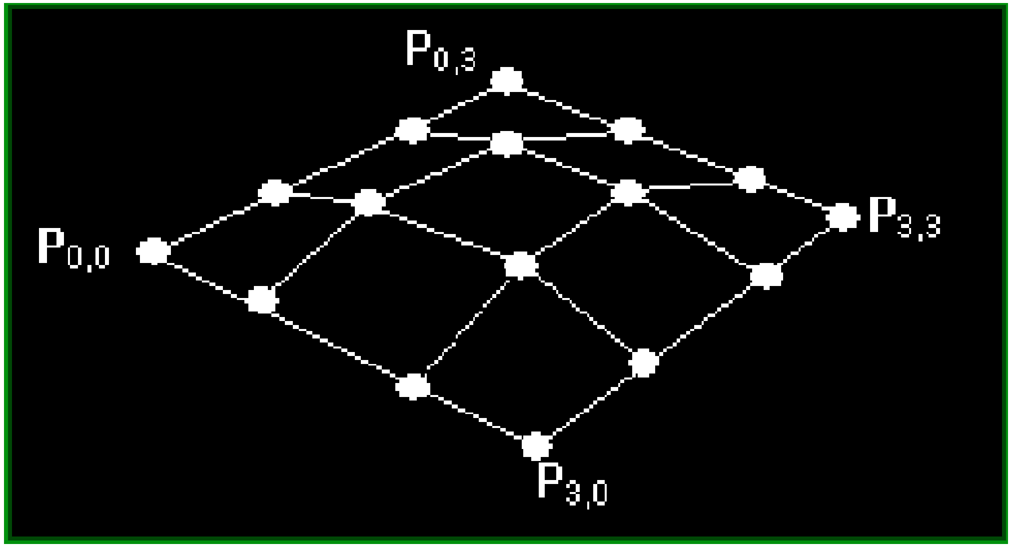
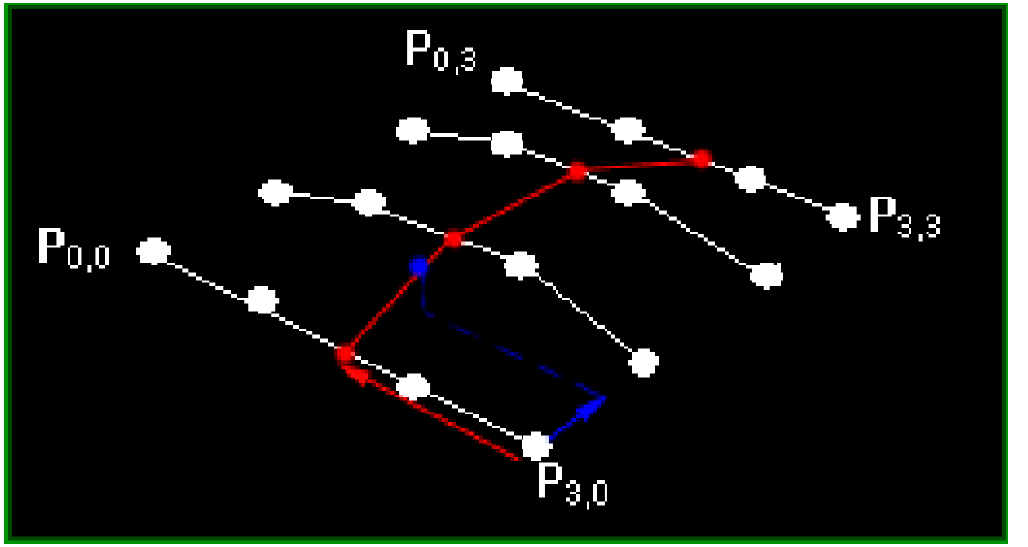
Bézier Surfaces

- In one dimension
  - $C_n(t) = \sum_{i=0}^{n} B_i^n(t)\, P_i, \quad t \in [0,1]$
- In two dimensions
  - $S_{n,m}(t,r) = \sum_{i=0}^{n} B_i^n(t) \sum_{j=0}^{m} B_j^m(r)\, P_i, \quad t,r \in [0,1]$
- P_i are points in 3D or 4D
- Convex linear combination of points P_i
  - Entire curve is in convex hull of points
  - Surface passes through 4 corner points
- Curve is smooth and differentiable

2D Cubic Bézier Surface

- 16 Control points
- Corner points set surface
- Interior points stretches surface





Bicubic Bézier Patch

$$
\begin{aligned}
P &= (1-v)^3 \Big((1-u)^3 P_{00} + 3(1-u)^2 u P_{01} + 3(1-u)u^2 P_{02} + u^3 P_{03}\Big) \\
&+ 3(1-v)^2 v \Big((1-u)^3 P_{10} + 3(1-u)^2 u P_{11} + 3(1-u)u^2 P_{12} + u^3 P_{13}\Big) \\
&+ 3(1-v)v^2 \Big((1-u)^3 P_{20} + 3(1-u)^2 u P_{21} + 3(1-u)u^2 P_{22} + u^3 P_{23}\Big) \\
&+ v^3 \Big((1-u)^3 P_{30} + 3(1-u)^2 u P_{31} + 3(1-u)u^2 P_{32} + u^3 P_{33}\Big) \\
&= (1-u)^3 \Big((1-v)^3 P_{00} + 3(1-v)^2 v P_{10} + 3(1-v)v^2 P_{20} + v^3 P_{30}\Big) \\
&+ 3(1-u)^2 u \Big((1-v)^3 P_{01} + 3(1-v)^2 v P_{11} + 3(1-v)v^2 P_{21} + v^3 P_{31}\Big) \\
&+ 3(1-u)u^2 \Big((1-v)^3 P_{02} + 3(1-v)^2 v P_{12} + 3(1-v)v^2 P_{22} + v^3 P_{32}\Big) \\
&+ u^3 \Big((1-v)^3 P_{03} + 3(1-v)^2 v P_{13} + 3(1-v)v^2 P_{23} + v^3 P_{33}\Big)
\end{aligned}
$$

Bicubic Bézier Patch Normal

$$
\begin{aligned}
\frac{\partial P}{\partial u} &= -3(1-u)^2 && \Big((1-v)^3 P_{00} + 3(1-v)^2 v P_{10} + 3(1-v)v^2 P_{20} + v^3 P_{30}\Big) \\
&+ 3(1-3u)(1-u) && \Big((1-v)^3 P_{01} + 3(1-v)^2 v P_{11} + 3(1-v)v^2 P_{21} + v^3 P_{31}\Big) \\
&+ 3u(2-3u) && \Big((1-v)^3 P_{02} + 3(1-v)^2 v P_{12} + 3(1-v)v^2 P_{22} + v^3 P_{32}\Big) \\
&+ 3u^2 && \Big((1-v)^3 P_{03} + 3(1-v)^2 v P_{13} + 3(1-v)v^2 P_{23} + v^3 P_{33}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P}{\partial v} &= -3(1-v)^2 && \Big((1-u)^3 P_{00} + 3(1-u)^2 u P_{01} + 3(1-u)u^2 P_{02} + u^3 P_{03}\Big) \\
&+ 3(1-3v)(1-v) && \Big((1-u)^3 P_{10} + 3(1-u)^2 u P_{11} + 3(1-u)u^2 P_{12} + u^3 P_{13}\Big) \\
&+ 3v(2-3v) && \Big((1-u)^3 P_{20} + 3(1-u)^2 u P_{21} + 3(1-u)u^2 P_{22} + u^3 P_{23}\Big) \\
&+ 3v^2 && \Big((1-u)^3 P_{30} + 3(1-u)^2 u P_{31} + 3(1-u)u^2 P_{32} + u^3 P_{33}\Big)
\end{aligned}
$$

$$
N = \frac{\partial P}{\partial u} \times \frac{\partial P}{\partial v}
$$

Surfaces in OpenGL

- Two-dimensional Evaluators
- Can be used to generate vertexes, normals, colors and textures
- Curve defined analytically using Bezier surfaces
- Evaluated at discrete points and rendered using polygons

Surfaces in OpenGL

- glEnable()
  - Enables types of data to generate
  - GL_AUTO_NORMAL generates normals for you
- glMap2d()
  - Defines control points and domain
- glEvalCoord2d()
  - Generates a data point
- glMapGrid2d() & glEvalMesh2()
  - Generates a series of data points

glMap2d(*type,Umin,Umax,Ustride,Uorder, Vmin,Vmax,Vstride,Vorder,points*)

- *type* of data to generate
  - GL_MAP1_VERTEX_[34]
  - GL_MAP1_NORMAL
  - GL_MAP1_COLOR_4
  - GL_MAP1_TEXTURE_COORD_[1-4]
- *Umin*&*Umax* and *Vmin*&Vmax are limits(often 0&1)
- *Ustride* is the number of values in data (3,4)
- *Vstride* is the number of values in a row of data
- *Uorder* & *Vorder* is the order of the curve (4=cubic)
- *points* is the array of data points (16 for bi-cubic)
- **Remember to also call glEnable()**

glEvalCoord2d(u,v)

- Generate one vertex for each glMap2d() type currently active (e.g. texture, normal, vertex)
- To generate the whole surface, loop over quads and call glEvalCoord2d() once for each vertex
- Exercise entire parameter space
  - u from Umin to Umax (0 to 1)
  - v from Vmin to Vmax (0 to 1)

Generating a complete surface

- glMapGrid2d(N , U1 , U2 , M , V1 , V2)
- glEvalMesh2(mode , N1 , N2 , M1 , M2)

- This is equivalent to

```
for (j=M1;j<M2;j++)
{
   glBegin(GL_QUAD_STRIP);
   for (i=N1;i<=N2;i++)
   {
      glEvalCoord1(U1+i*(U2-U1)/N , V1+j*(V2-V1)/M);
      glEvalCoord1(U1+i*(U2-U1)/N , V1+(j+1)*(V2-V1)/M);
   }
   glEnd();
}
```

The Utah Teapot

- Generated by Martin Newell in 1975
  - 32 Patches specified as Bezier surfaces
  - 10 Base patches with reflections
  - 126 control points
- Complex shape
  - Hole in handle
  - Hollow spout
- Non-convex
  - Can cast shadows on itself

The Utah Teapot: Then and Now