Ray Tracing: Special Topics CSCI 4239/5239 Advanced Computer Graphics Spring 2023

Theoretical foundations

Ray Tracing from the Ground Up Chapters 13-15

- Bidirectional Reflectance Distribution Function
 - BRDF
 - Describes how light is reflected on each bounce
 - Chains to transfer colors



Figure 1. The ray-tracing process.

Radiometric Quantities

- Radiant Energy Q (J)
- Radiant Flux $\phi = dQ/dt$ (W)
- Radiant Flux Density $d\phi/dA$ (W/m²)
- Irradiance E [Arriving flux density]
- Radiant exitance M [Leaving flux density]
- Radiant Intensity I $d\phi/d\omega$ (W/m²/sr)
- Radiance L $d^2 \phi/dAd\omega$ (W/m²/sr)

Ray Properties

- Radiance is constant along rays
- Radiance can be defined at the eye
- Radiance can be defined at a point



Figure 13.1. (a) Radiant flux in a cone of incident angles $d\omega$ passing through a surface element dA^{\perp} . (b) In the limit $d\omega \rightarrow 0$ and $dA \rightarrow 0$, the radiance is defined as coming from a single direction ω . The point *p* can be an arbitrary point in space.

Angular Dependence on Irradiance

- Lambert's Law
 - $-L = d^2 \phi / dA \cdot d\omega \cdot \cos \theta$



Figure 13.2. (a) and (b) Irradiance spreads out over a larger area as the incidence angle θ increases. (c) An enlarged view of the incident beam.

Notation and Directions



Figure 13.3. The incoming direction ω_i and reflected direction ω_o point away from the surface and are on the same side of the surface as the normal. Each direction is defined by its polar and azimuth angles (θ , ϕ). These are arbitrary directions; for perfect mirror reflection, $\phi_o = \phi_i \pm \pi$, as illustrated in Figure 24.2(b).

BRDF

- Definition
 - $f(p, \omega_i, \omega_o) = dL_o(p, \omega) / dL_i(p, \omega) \cos \theta_i d\omega_i$
- Properties
 - Reciprocity
 - $f(p, \omega_i, \omega_o) = f(p, \omega_o, \omega_i)$
 - Linearity
 - Sum all BRDFs at a point
 - Conservation of energy
 - Total re-radiated energy must be less than incident

Common BDRFs

• Diffuse $f(p, \omega_i, \omega_o) = M_d(p)$



Figure 13.6. Light being scattered from a perfectly diffuse surface.

• Specular $f(p, \omega_i, \omega_o) = M_s(p) (R \cdot \omega_o)^s$ - R = 2(N · L)N-L



Bouncing Rays from Surfaces



Figure 14.4. (a) Mirror reflection can be modeled by tracing a single reflected ray at each hit point; (b) modeling glossy specular light transport between surfaces requires many rays to be traced per pixel; (c) modeling perfect diffuse light transport between surfaces also requires many rays to be traced per pixel.

Antialiasing



Figure 4.4. Shaded sphere: (a) one sample per pixel; (b) 16 samples per pixel; (c) enlarged view of top-right section of (b).

Super-sampling Pixels



Figure 4.7. (a) 25 random samples in a pixel; (b) 25 jittered samples; (c) same as (b) but with sub-grid lines shown.

Super-sampling Area Lights



Figure 5.2. Shading a surface with an area light and four samples per pixel.

Side-effects of Sampling Pattern



Figure 5.6. Global illumination images that exhibit bad aliasing caused by using the same samples in vertical columns (a) and in a regular horizontal displacement (b).

Depth of Field

- Important for realism
 - Background is "fuzzy"
- Partly out of focus
 - Imperfect optics
 - Turbulence
- Graphic backgrounds are often too perfect



Thin Lens Theory



Figure 10.1. Cross section through a thin lens showing a focal plane and its corresponding image plane.

Out of Focus Images



Figure 10.2. Rays starting a point *q* go through the image plane of *p* at different locations, with the result that *q* will appear out of focus.

Depth of Field Results



Figure 10.9. (a) When the lens radius is zero, the image is the same as a pinhole-camera image with everything in focus; (b) noisy image from using one random sample per pixel.



Figure 10.12. Mirrored surface.

Ambient Occlusion

- Floor has a vague shadow outline
- Parts of object near floor is darker
- Ambient light is not anisotropic and uniform



Image courtesy of Mark Howard, Stanford bunny model courtesy of Greg Turk and the Stanford University Graphics Laboratory

Computing Ambient Occlusion



Figure 17.1. Point *a* on the sphere receives the maximum amount of ambient light because the box isn't visible; point *b* doesn't receive the maximum amount because the box blocks some of the incoming ambient light.



Figure 17.2. Various hit points on the plane and the sphere, with sample shadow rays.

Ambient Occlusion Results



Figure 17.12. Bunny scene rendered with 256 samples per pixel: (a) min_amount = 1; (b) min_amount = 0.25; (c) min_amount = 0.



Figure 17.13. Random boxes rendered with 64 samples per pixel: (a) min_amount = 1.0; (b) min_amount = 0.25.

Mirror Reflection

- Mirror reflections are a signature of ray tracing
 - Shiny objects
 - Glass
 - Metal
 - Multiple reflections may occur
- Occurs naturally in ray tracing
- Requires tracing ray through multiple bounces
- Adds significant effort



Reflective spheres by Burt Flugleman, Rundle Street Mall, Adelaide. Photograph by Kevin Suffern.

Conservation of Energy

- Mirrors reflect almost all the energy
- Retains beam geometry



Figure 24.3. When a beam of light is reflected from a perfect mirror, its cross section area is unchanged after reflection and is therefore independent of the angle of incidence θ_i .

Number of Reflections

- 0 dull
- 1 "simple" mirror
- >1 "hall of mirrors"
- Effort grows with number of bounces





Figure 24.7. (a) The scene from Figure 24.6 rendered with max_depth = 10; (b) close-up view of the yellow-green sphere and the cylinder from a different viewpoint than in (a).

Hall of Mirrors (Showcases Ray Tracing)



Figure 24.18. (a) Hall of mirrors with max_depth = 19; (b) close-up view of the multiple reflections between the floor mirror and the sphere.

Mirror vs Glossy Reflection

- Mirror reflections are "perfect"
- Glossy reflections are "imperfect"
 - Reflected ray = $2(N \cdot V)N V + \epsilon$
 - Super-sample for many values of $\boldsymbol{\epsilon}$



Figure 24.8. The green sphere has glossy specular highlights and perfect mirror reflections (a) and glossy reflections (b).

Degrees of perfection



Figure 25.8. Glossy sphere surrounded by the Uffizi image and rendered with the following values of *e*: (a) 1.0; (b) 10.0; (c) 100.0; (d) 1000.0; (e) 10000.0; (f) 100000.0.

Simple Transparency

- Light passes through objects
- Light changes through object
 - Rays are bent
 - Colors are changed
- Rays multiply
 - Reflected
 - Transmitted



Refraction

- Index of refraction $\eta = c/v$
 - Vacuum 1
 - Air 1.0003
 - Water 1.33
 - Glass 1.5
 - Diamond 2.42
- Snell's law
 - $-\sin\theta_i/\sin\theta_t = \eta$



Figure 27.2. Reflected and transmitted rays at the boundary between two transparent media.

Media Transitions

- Direction of bend depends on whether the refrection index increases or decreases
 - Air $\boldsymbol{\eta}$ is very low
 - Angles decrease intc liquids
 - Angles increase out of liquids



Figure 27.3. Direction change of *t* when $\eta > 1$.



Figure 27.4. Direction change of *t* when $\eta < 1$.

Internal reflections

- Critical angle
 - Refraction bends ray back into medium
- Higher η contrast causes larger critical angle
 - That is why diamonds are so sparkly



Figure 27.5. Total internal reflection: (a) and (b) $\theta_i < \theta_c$; (c) $\theta_i = \theta_c$; (d) $\theta_i > \theta_c$.

Transparency require bifurcating rays



Figure 27.6. Transparent objects with reflected and transmitted rays.



Figure 27.7. The ray tree that corresponds to Figure 27.6.

Objects Appearance

- Object inside other material
 - Objects are magnified when not viewed parallel to the normal
 - Object's apparent position is displaced
- Objects on other side
 - Objects apparent position is displaced



Figure 27.8. The angle of a differential cone of incident radiance changes as it crosses the boundary between two dielectrics.



Figure 27.9. Ray and radiance-transfer directions through a transparent object.

Distortion by Glass Spheres

Sphere as a lens



Figure 27.22. Transparent sphere in front of text.

 Eye position is critical



Figure 27.23. A transparent sphere with the camera far from the sphere (a) and close to the sphere (b).

Light movement through sphere

Magnification





Internal reflection



Figure 27.11. Reflected and transmitted rays generated by a ray r_0 that hits a unit sphere with impact parameter *b*. The lengths of the (unit) normals and the sphere are not drawn on the same scale.

Realistic Transparency

- Three η 's
 - Air
 - Glass
 - Water
- Colored liquid
- Beveled edges
 - Glass
 - Meniscus
- Mixed transparency
 - Foam is opaque



Image courtesy of John Avery

Reflectance and Attenuation



Figure 28.4. Radiance attenuation in a dielectric.
Multiple Internal Reflections



Figure 28.19. A transparent box with multiple reflected and transmitted rays.



Figure 28.12. (a) Stanford bunny rendered with $c_f = (0.65, 0.45, 0)$ and max_depth = 2; (b) max_depth = 10; (c) horse model rendered with $c_f = (0.65, 0.65, 0.1)$ and max_depth = 10.

Colored Beaker



Figure 28.37. A more sophisticated glass of water has a curved top, rounded edges, and a meniscus for the water.



Figure 28.38. Glass of water and straw rendered with: (a) no shadows; (b) camera looking up; (c) shadows and direct illumination on the straw.

The Fish Bowl

- Making it real
 - Complex shape
 - Three media
 - Colored media
 - Beveled edges
- Challenges
 - Multiple reflections
 - Refraction







Figure 28.40. (a) Construction of the rim; (b) construction of the meniscus.



Adding Textures

- Per pixel modification of surface appearance
- Use texture coordinates to map textures to objects
 - When ray tracing, you have to do this yourself
- Textures modify ray color on each bounce



Figure 29.1. Interior scene rendered with no textures.



Figure 29.2. Same scene as in Figure 29.1 but rendered with a variety of textures. The water surface is Ken Musgrave's water bump map, as described in Musgrave (2003b).

Caustics







Tim Dunn's Gallery











Production Ray Tracing Tim Dunn



Raw Render Pass



Ambient Occculsion Pass

Luminosity Pass



Shadow Pass



Diffuse Pass



Dissue Lighting Pass







Reflections in Raster Methods

- Two possible approaches
 - Textures (image space)
 - Virtual objects (object space)
- Both approaches requires rendering the scene multiple times
- Mirrors can be planar or curved
- Mirrors are "windows" to the virtual scene

Virtual Objects

- Draw object where they seem to appear
- Clip to reflector



Start with scene

Create virtual object

Clip to reflector

Planar Reflection Equation

- Point on mirror P
- Normal vector V

$$R = \begin{pmatrix} 1 - 2V_x^2 & -2V_xV_y & -2V_xV_z & 2(P \cdot V)V_x \\ -2V_xV_y & 1 - 2V_y^2 & -2V_yV_z & 2(P \cdot V)V_y \\ -2V_xV_z & -2V_yV_z & 1 - 2V_z^2 & 2(P \cdot V)V_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rendering Order

- Reflections are difficult when the mirror is an object inside the scene
 - Mirror on wall is easier



Draw unreflected objects first (except reflector)



Draw virtual objects clipped by depth test



Draw reflector last

Figure 17.3 Masking reflections with depth buffering.

Limiting the Reflector

- User defined clipping volume
 - Front and back clipping planes
 - Frustrum
- Stencil buffer
- Special cases
 - Scissors test
 - Alpha blending

Reflections using Textures

- Quads
 - Simple mirrors
- Environment maps
 - Cube map
 - Sphere map







Figure 17.4 Masking reflections using projective texture.

Reflections from Curved Surfaces

- Cannot be done using virtual objects
- Readily done by distorting textures



Figure 17.5 Normals and reflection vectors in curved reflectors.

Inter-reflections

- Hall of mirrors requires multiple passes
 - Similar to max-levels



Figure 17.13 Clipping multiple interreflections with stencil.