# Ray Tracing: Special Topics CSCI 4239/5239 <br> Advanced Computer Graphics Spring 2024 

## Theoretical foundations

## Ray Tracing from the Ground Up Chapters 13-15

- Bidirectional Reflectance Distribution Function
- BRDF
- Describes how light is reflected on each bounce
- Chains to transfer colors


Figure 1. The ray-tracing process.

## Radiometric Quantities

- Radiant Energy Q (J)
- Radiant Flux $\phi=d Q / d t(\mathrm{~W})$
- Radiant Flux Density $d \phi / d A\left(W / m^{2}\right)$
- Irradiance E [Arriving flux density]
- Radiant exitance M [Leaving flux density]
- Radiant Intensity I $d \phi / d \omega$ (W/m²/sr)
- Radiance L $d^{2} \phi / d A d \omega\left(W / m^{2} / s r\right)$


## Ray Properties

- Radiance is constant along rays
- Radiance can be defined at the eye
- Radiance can be defined at a point


Figure 13.1. (a) Radiant flux in a cone of incident angles $d \omega$ passing through a surface element $d A^{\perp}$. (b) In the limit $d \omega \rightarrow 0$ and $d A \rightarrow 0$, the radiance is defined as coming from a single direction $\omega$. The point $p$ can be an arbitrary point in space.

## Angular Dependence on Irradiance

- Lambert's Law
$-L=d^{2} \phi / d A \cdot d \omega \cdot \cos \theta$

(a)

(b)

Figure 13.2. (a) and (b) Irradiance spreads out over a larger area as the incidence angle $\theta$ increases. (c) An enlarged view of the incident beam.

## Notation and Directions



Figure 13.3. The incoming direction $\omega_{\mathrm{i}}$ and reflected direction $\omega_{\mathrm{o}}$ point away from the surface and are on the same side of the surface as the normal. Each direction is defined by its polar and azimuth angles $(\theta, \phi)$. These are arbitrary directions; for perfect mirror reflection, $\phi_{0}=\phi_{i} \pm \pi$, as illustrated in Figure 24.2(b).

## BRDF

- Definition
$-f\left(p, \omega_{i}, \omega_{o}\right)=d L_{o}(p, \omega) / d L_{i}(p, \omega) \cos \theta_{i} d \omega_{i}$
- Properties
- Reciprocity
- $\mathrm{f}\left(\mathrm{p}, \omega_{\mathrm{i}}, \omega_{0}\right)=\mathrm{f}\left(p, \omega_{0}, \omega_{\mathrm{i}}\right)$
- Linearity
- Sum all BRDFs at a point
- Conservation of energy
- Total re-radiated energy must be less than incident


## Common BDRFs

- Diffuse $\mathrm{f}\left(p, \omega_{i}, \omega_{o}\right)=M_{d}(p)$


Figure 13.6. Light being scattered from a perfectly diffuse surface.

- Specular $f\left(p, \omega_{i}, \omega_{0}\right)=M_{s}(p)\left(R \cdot \omega_{0}\right)^{s}$
$-\mathrm{R}=2(\mathrm{~N} \cdot \mathrm{~L}) \mathrm{N}-\mathrm{L}$

perfect specular reflection
(a)

(b)

Figure 14.3. (a) Perfect specular reflection; (b) glossy specular reflection.

## Bouncing Rays from Surfaces


(a)

(b)

(c)

Figure 14.4. (a) Mirror reflection can be modeled by tracing a single reflected ray at each hit point; (b) modeling glossy specular light transport between surfaces requires many rays to be traced per pixel; (c) modeling perfect diffuse light transport between surfaces also requires many rays to be traced per pixel.

## Antialiasing


(a)

(b)

(c)

Figure 4.4. Shaded sphere: (a) one sample per pixel; (b) 16 samples per pixel; (c) enlarged view of top-right section of (b).

## Super-sampling Pixels



Figure 4.7. (a) 25 random samples in a pixel; (b) 25 jittered samples; (c) same as (b) but with sub-grid lines shown.

## Super-sampling Area Lights



Figure 5.2. Shading a surface with an area light and four samples per pixel.

## Side-effects of Sampling Pattern



Figure 5.6. Global illumination images that exhibit bad aliasing caused by using the same samples in vertical columns (a) and in a regular horizontal displacement (b).

## Depth of Field

- Important for realism - Background is "fuzzy"
- Partly out of focus
- Imperfect optics
- Turbulence
- Graphic backgrounds are often too perfect



## Thin Lens Theory



Figure 10.1. Cross section through a thin lens showing a focal plane and its corresponding image plane.

## Out of Focus Images



Figure 10.2. Rays starting a point $q$ go through the image plane of $p$ at different locations, with the result that $q$ will appear out of focus.

## Depth of Field Results



Figure 10.9. (a) When the lens radius is zero, the image is the same as a pinhole-camera image with everything in focus; (b) noisy image from using one random sample per pixel.


Figure 10.12. Mirrored surface.

## Ambient Occlusion

- Floor has a vague shadow outline
- Parts of object near floor is darker
- Ambient light is not anisotropic and uniform


Image courtesy of Mark Howard, Stanford bunny model courtesy of Greg Turk and the Stanford

University Graphics Laboratory

## Computing Ambient Occlusion



Figure 17.1. Point $a$ on the sphere receives the maximum amount of ambient light because the box isn't visible; point $b$ doesn't receive the maximum amount because the box blocks some of the incoming ambient light.


Figure 17.2. Various hit points on the plane and the sphere, with sample shadow rays.

## Ambient Occlusion Results



Figure 17.12. Bunny scene rendered with 256 samples per pixel: (a) min_amount $=1$; (b) min_amount $=0.25$; (c) min_amount $=0$.

(a)

(b)

Figure 17.13. Random boxes rendered with 64 samples per pixel: (a) min_amount $=1.0$; (b) min_amount $=0.25$.

## Mirror Reflection

- Mirror reflections are a signature of ray tracing
- Shiny objects
- Glass
- Metal
- Multiple reflections may occur
- Occurs naturally in ray tracing
- Requires tracing ray through multiple bounces
- Adds significant effort


## Conservation of Energy

- Mirrors reflect almost all the energy
- Retains beam geometry

(a)

(b)

Figure 24.3. When a beam of light is reflected from a perfect mirror, its cross section area is unchanged after reflection and is therefore independent of the angle of incidence $\theta_{\mathrm{i}}$.

## Number of Reflections

- 0 dull
- 1 "simple" mirror
- >1 "hall of mirrors"
- Effort grows with number of bounces


Figure 24.6. A scene rendered with max_depth $=0$ (a) and max_depth $=1$ (b).


Figure 24.7. (a) The scene from Figure 24.6 rendered with max_depth $=10$; (b) close-up view of the yellow-green sphere and the cylinder from a different viewpoint than in (a).

## Hall of Mirrors <br> (Showcases Ray Tracing)


(a)

(b)

Figure 24.18. (a) Hall of mirrors with max_depth $=19$; (b) close-up view of the multiple reflections between the floor mirror and the sphere.

## Mirror vs Glossy Reflection

- Mirror reflections are "perfect"
- Glossy reflections are "imperfect"
- Reflected ray $=2(\mathrm{~N} \cdot \mathrm{~V}) \mathrm{N}-\mathrm{V}+\varepsilon$
- Super-sample for many values of $\varepsilon$




## Degrees of perfection


(a)

(d)

(b)

(e)

(c)

(f)

Figure 25.8. Glossy sphere surrounded by the Uffizi image and rendered with the following values of $e$ : (a) 1.0; (b) 10.0; (c) 100.0; (d) 1000.0; (e) 10000.0; (f) 100000.0.

## Simple Transparency

- Light passes through objects
- Light changes through object
- Rays are bent
- Colors are changed
- Rays multiply
- Reflected
- Transmitted



## Refraction

- Index of refraction $\eta=c / v$
- Vacuum 1
- Air 1.0003
- Water 1.33
- Glass 1.5
- Diamond 2.42


Figure 27.2. Reflected and transmitted rays at the boundary between two transparent
media.

- Snell's law
$-\sin \theta_{\mathrm{i}} / \sin \theta_{\mathrm{t}}=\eta$


## Media Transitions

- Direction of bend depends on whethes the refrection index increases or decreases
- Air $\eta$ is very low
- Angles decrease intc liquids
- Angles increase out of liquids


Figure 27.3. Direction change of $t$ when $\eta>1$.


Figure 27.4. Direction change of $t$ when $\eta<1$.

## Internal reflections

- Critical angle
- Refraction bends ray back into medium
- Higher $\eta$ contrast causes larger critical angle
- That is why diamonds are so sparkly

(a)

(c)

(b)

(d)


## Transparency require bifurcating rays



Figure 27.6. Transparent objects with reflected and transmitted rays.


Figure 27.7. The ray tree that corresponds to Figure 27.6.

## Objects Appearance

- Object inside other material
- Objects are magnified when not viewed parallel to the normal
- Object's apparent position is displaced
- Objects on other side
- Objects apparent position is displaced


Figure 27.8. The angle of a differential cone of incident radiance changes as it crosses the boundary between two dielectrics.


Figure 27.9. Ray and radiance-trans-
fer directions through a transparent
object.

## Distortion by Glass Spheres

- Sphere as a lens


Figure 27.22. Transparent sphere in front of text.

- Eye position is critical


Figure 27.23. A transparent sphere with the camera far from the sphere (a) and close to the sphere (b).

## Light movement through sphere

- Magnification


Figure 27.17. Reflected and transmitted rays generated by a ray $r_{0}$ that hits a unit sphere with impact parameter $b$, where the sphere has $\eta<1$.

- Internal reflection


Figure 27.11. Reflected and transmitted rays generated by a ray $r_{0}$ that hits a unit sphere with impact parameter $b$. The lengths of the (unit) normals and the sphere are not drawn on the same scale.

## Realistic Transparency

- Three $\eta$ 's
- Air
- Glass
- Water
- Colored liquid
- Beveled edges
- Glass
- Meniscus
- Mixed transparency


Image courtesy of John Avery

- Foam is opaque


## Reflectance and Attenuation



Figure 28.4. Radiance attenuation in a dielectric.

## Multiple Internal Reflections



Figure 28.19. A transparent box with multiple reflected and transmitted rays.


Figure 28.12. (a) Stanford bunny rendered with $\boldsymbol{c}_{\mathrm{f}}=(0.65,0.45,0)$ and max_depth $=2$;
(b) max_depth $=10$; (c) horse model rendered with $c_{i}=(0.65,0.65,0.1)$ and max_depth $=10$.

## Colored Beaker



Figure 28.37. A more sophisticated glass of water has a curved top, rounded edges, and a meniscus for the water.

(a)

(b)

(c)

Figure 28.38. Glass of water and straw rendered with: (a) no shadows; (b) camera looking up; (c) shadows and direct illumination on the straw.

## The Fish Bowl

- Making it real
- Complex shape
- Three media
- Colored media
- Beveled edges
- Challenges
- Multiple reflections
- Refraction

(a)

(b)

Figure 28.39. (a) Basic fishbowl; (b) fishbowl with flat base and meniscus,

(a)

(b)

Figure 28.40. (a) Construction of the rim; (b) construction of the meniscus.


## Adding Textures

- Per pixel modification of surface appearance
- Use texture coordinates to map textures to objects
- When ray tracing, you have to do this yourself
- Textures modify ray color on each bounce


Figure 29.1. Interior scene rendered with no textures.


## Caustics



## Tim Dunn's Gallery






## Production Ray Tracing Tim Dunn



Raw Render Pass



Ambient Occculsion Pass

Luminosity Pass


Shadow Pass


Diffuse Pass


Dissue Lighting Pass

Specular Color Pass


Final Composite

## Reflections in Raster Methods

- Two possible approaches
- Textures (image space)
- Virtual objects (object space)
- Both approaches requires rendering the scene multiple times
- Mirrors can be planar or curved
- Mirrors are "windows" to the virtual scene


## Virtual Objects

- Draw object where they seem to appear
- Clip to reflector


Start with scene


Create virtual object


Clip to reflector

## Planar Reflection Equation

- Point on mirror $P$
- Normal vector $V$

$$
R=\left(\begin{array}{cccc}
1-2 V_{x}^{2} & -2 V_{x} V_{y} & -2 V_{x} V_{z} & 2(P \cdot V) V_{x} \\
-2 V_{x} V_{y} & 1-2 V_{y}^{2} & -2 V_{y} V_{z} & 2(P \cdot V) V_{y} \\
-2 V_{x} V_{z} & -2 V_{y} V_{z} & 1-2 V_{z}^{2} & 2(P \cdot V) V_{z} \\
0 & 0 & 0 & 1
\end{array}\right)
$$

## Rendering Order

- Reflections are difficult when the mirror is an object inside the scene
- Mirror on wall is easier


Draw unreflected objects first (except reflector)


Draw virtual objects clipped by depth test


Draw reflector last

Figure 17.3 Masking reflections with depth buffering.

## Limiting the Reflector

- User defined clipping volume
- Front and back clipping planes
- Frustrum
- Stencil buffer
- Special cases
- Scissors test
- Alpha blending


## Reflections using Textures

- Quads
- Simple mirrors
- Environment maps
- Cube map
- Sphere map



## Reflections from Curved Surfaces

- Cannot be done using virtual objects
- Readily done by distorting textures


Figure 17.5 Normals and reflection vectors in curved reflectors.

## Inter-reflections

- Hall of mirrors requires multiple passes
- Similar to max-levels


Figure 17.13 Clipping multiple interreflections with stencil.

