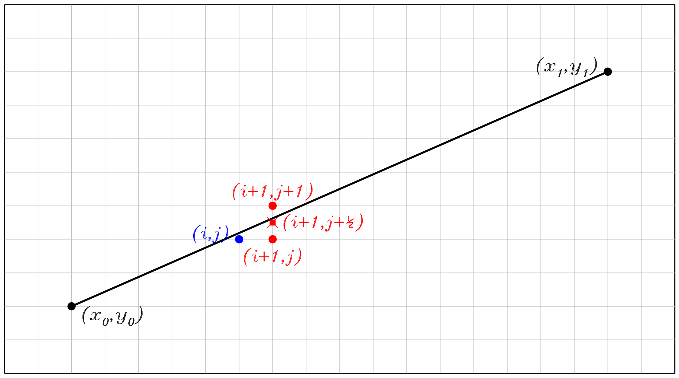
Scan Conversion with Bresenham Algorithm

CSCI 4229/5229 Summer 2019



Let $\delta x = x_1 - x_0$ and $\delta y = y_1 - y_0$, so that

$$y = \frac{\delta y}{\delta x}x + c, \qquad c = y_0 - \frac{\delta y}{\delta x}x_0.$$

Multiply by $2\delta x$:

$$2y\delta x = 2x\delta y + 2c\delta x$$

Let $K = 2c\delta x$ and define

$$F(x,y) = 2x\delta y - 2y\delta x + K$$

When $F(x,y) = 0$ then (x,y) is on the line.
When $F(x,y) > 0$ then (x,y) is below the line.
When $F(x,y) < 0$ then (x,y) is above the line.

$$\begin{aligned}
F(i,j) &= 2i\delta y - 2j\delta x + K \\
F(i+1,j) &= 2(i+1)\delta y - 2j\delta x + K \\
&= 2i\delta y - 2j\delta x + K + 2\delta y \\
&= F(i,j) + 2\delta y \\
F(i+1,j+\tfrac{1}{2}) &= 2(i+1)\delta y - 2(j+\tfrac{1}{2})\delta x + K \\
&= 2i\delta y - 2j\delta x + K + 2\delta y - \delta x \\
&= F(i,j) + 2\delta y - \delta x \\
F(i+1,j+1) &= 2(i+1)\delta y - 2(j+1)\delta x + K \\
&= 2i\delta y - 2j\delta x + K + 2\delta y - 2\delta x \\
&= F(i,j) + 2\delta y - 2\delta x
\end{aligned}$$

Start with $F(x_0,y_0) = 0$

Evaluate $F(i+1,j+\frac{1}{2}) = F(i,j) + 2\delta y - \delta x$.

If $F(i+1,j+\frac{1}{2}) > 0$, the midpoint is below the line, so the next pixel is $(i+1,j+1)$, and $F(i+1,j+1) = F(i,j) + 2\delta y - 2\delta x$.

Otherwise the midpoint is above the line, so the next pixel is $(i+1,j)$, and $F(i+1,j) = F(i,j) + 2\delta y$.