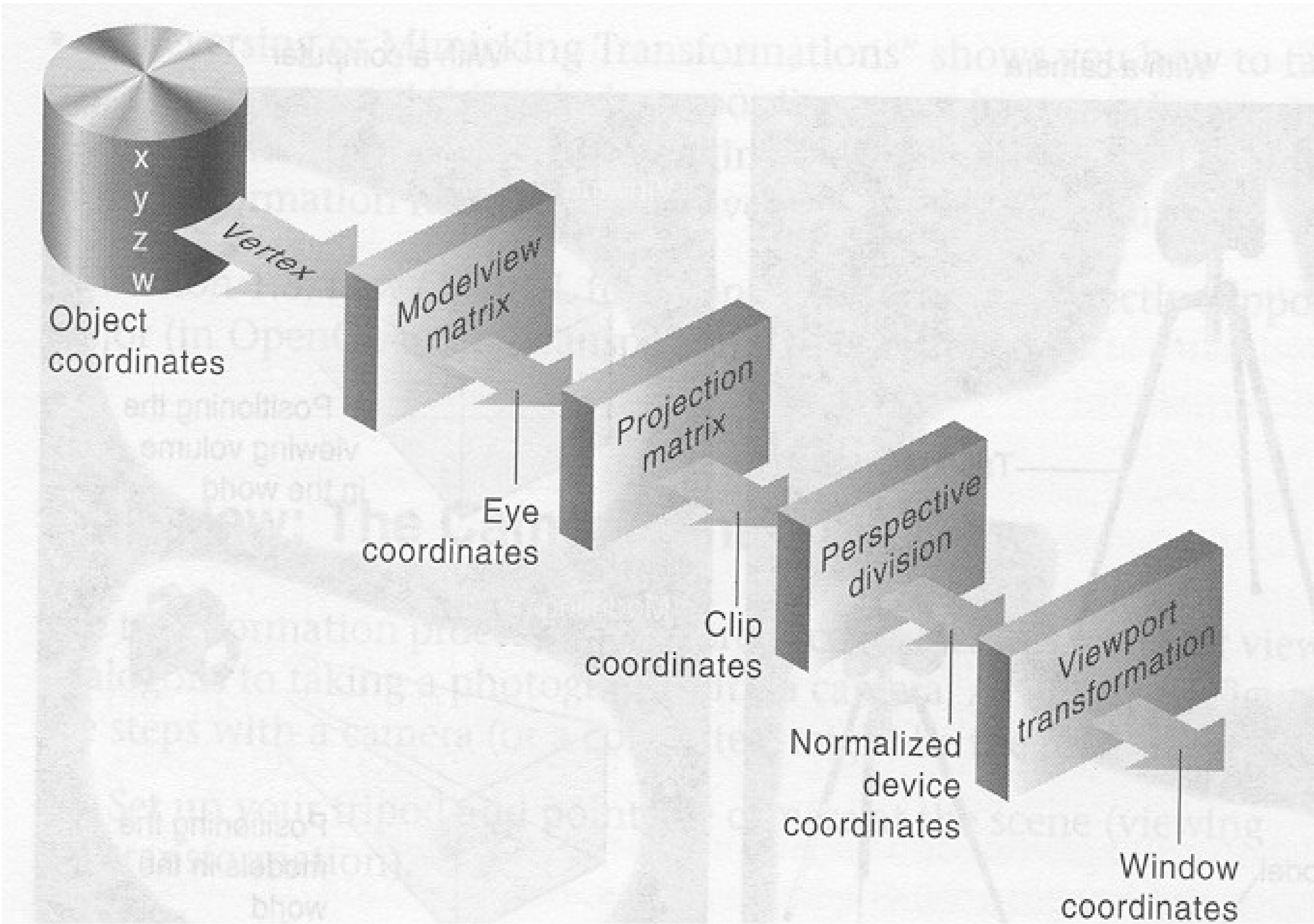
Projections

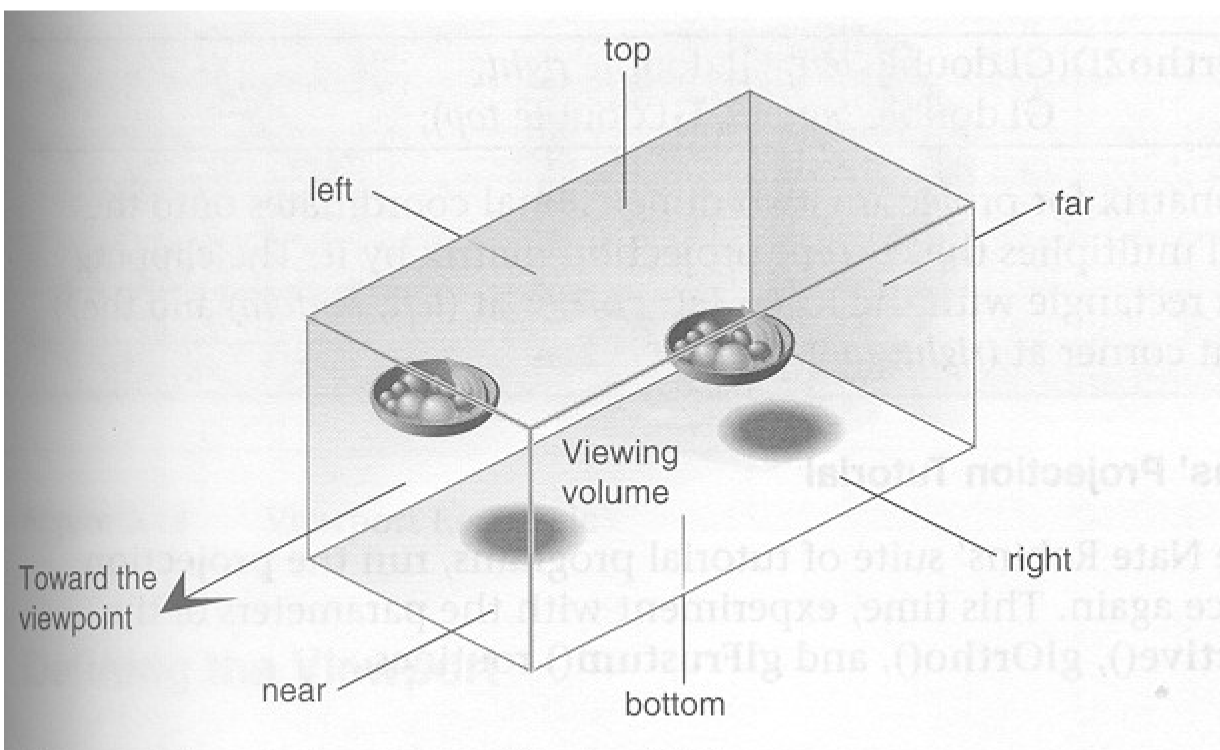
CSCI 4229/5229
Computer Graphics
Fall 2006

OpenGL Transformation Pipeline



Parallel Projection

- Apply rotation matrix to map direction of projection to *Z* axis and up to *Y* axis
- Scale to canonical volume



From: *OpenGL Red Book*

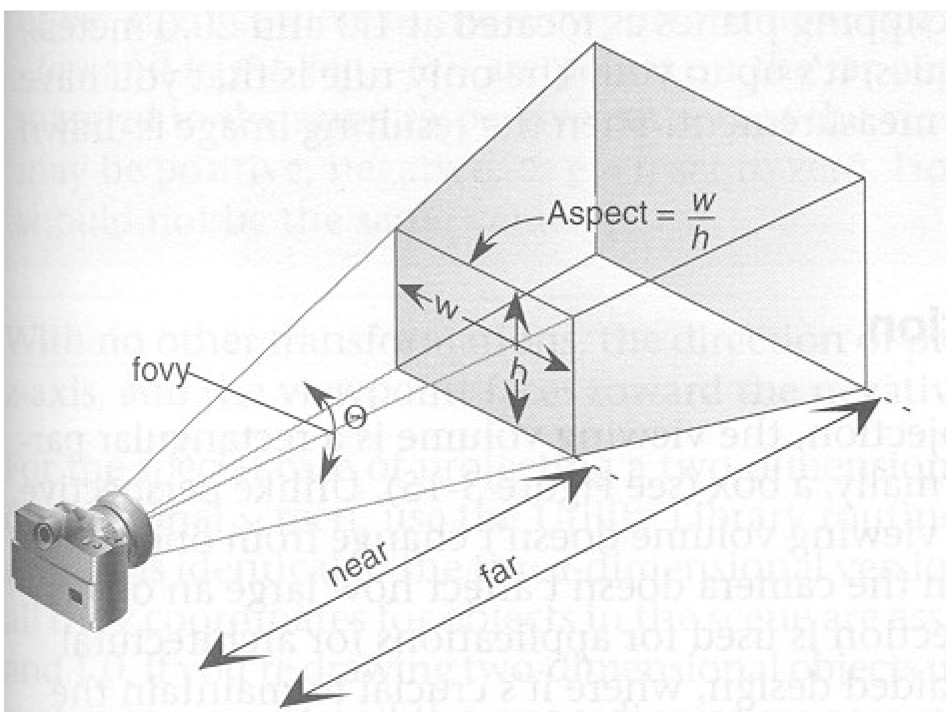
glOrtho(x_{min}, x_{max}, y_{min}, y_{max}, z_{min}, z_{max})

glOrtho Projection Matrix

$$\begin{pmatrix} \frac{2}{x_{max}-x_{min}} & 0 & 0 & \frac{x_{max}+x_{min}}{x_{max}-x_{min}} \\ 0 & \frac{2}{y_{max}-y_{min}} & 0 & \frac{y_{max}+y_{min}}{y_{max}-y_{min}} \\ 0 & 0 & \frac{2}{z_{max}-z_{min}} & \frac{z_{max}+z_{min}}{z_{max}-z_{min}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Perspective Transformation

- Apply rotation matrix to map eye position to center of scene to negative *Z* and up to *Y* axes
- Scale *(x,y)* inversely proportional to distance
- Scale to canonical volume



From: *OpenGL Red Book*

gluPerspective(fovy,aspect,Znear,Zfar)

- *fovy* is the angle in the up/down direction
- *aspect* is is the horizontal to vertical ratio
- *Znear* is the distance to the near clipping plane
  - Killer fact Znear > 0
- *Zfar* is the distance to the far clipping plane
  - *Zfar* > *Znear*
- *Zfar-Znear* determines *Z* resolution since the Z buffer has finite precision

gluPerspective(fovy,aspect,Znear,Zfar)

Let θ = fovy/2

gluPerspective Projection Matrix

$$\begin{pmatrix} \frac{\cot\theta}{aspect} & 0 & 0 & 0 \\ 0 & \cot\theta & 0 & 0 \\ 0 & 0 & \frac{z_{far}+z_{near}}{z_{far}-z_{near}} & \frac{2z_{far}z_{near}}{z_{far}-z_{near}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

gluLookAt(E_x, E_y, E_z , C_x, C_y, C_z , U_x, U_y, U_z)

- (E_x, E_y, E_z) is the eye position
- (C_x, C_y, C_z) is the position you look at
- (U_x, U_y, U_z) is the up direction
- C-E determines the distance in the Z direction
- The Z distance to each object (from E) determines the reduction in the (x,y) direction

gluLookAt(E_x,E_y,E_z , C_x,C_y,C_z , U_x,U_y,U_z)

$$\begin{array}{ll} \text{Forward} & \mathbf{F} = \mathbf{C} - \mathbf{E} \\ \text{Sideways} & \mathbf{S} = \mathbf{F} \times \mathbf{U} \\ \text{Up} & \mathbf{U} = \mathbf{S} \times \mathbf{F} \end{array}$$

$$\begin{aligned} \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} &= \begin{pmatrix} S_x & U_x & -F_x & 0 \\ S_y & U_y & -F_y & 0 \\ S_z & U_z & -F_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -E_x \\ 0 & 1 & 0 & -E_y \\ 0 & 0 & 1 & -E_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \\ &= \begin{pmatrix} S_x & U_x & -F_x & -E_xS_x - E_yU_x + E_zF_x \\ S_y & U_y & -F_y & -E_xS_y - E_yU_y + E_zF_y \\ S_z & U_z & -F_z & -E_xS_z - E_yU_z + E_zF_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \end{aligned}$$