

Homogeneous Coordinates

**CSCI 4229/5229
Computer Graphics
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2D Homogeneous Transformations

Translation:

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & \delta x \\ 0 & 1 & \delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Scaling:

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Rotation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

3D Homogeneous Translation and Scaling

Translation:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Scaling:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

3D Homogeneous Rotation

Rotation around the z axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotation around the x axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotation around the y axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

General Homogeneous Rotation

Rotation around a unit vector (X, Y, Z) :

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 + (1 - \cos \theta)(X^2 - 1) & -Z \sin \theta + (1 - \cos \theta)XY & Y \sin \theta + (1 - \cos \theta)XZ & 0 \\ Z \sin \theta + (1 - \cos \theta)XY & 1 + (1 - \cos \theta)(Y^2 - 1) & -X \sin \theta + (1 - \cos \theta)YZ & 0 \\ -Y \sin \theta + (1 - \cos \theta)XZ & X \sin \theta + (1 - \cos \theta)YZ & 1 + (1 - \cos \theta)(Z^2 - 1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotate x axis to unit vector (X_0, Y_0, Z_0) and y axis to unit vector (X_1, Y_1, Z_1) :

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} X_0 & X_1 & Y_0Z_1 - Y_1Z_0 & 0 \\ Y_0 & Y_1 & Z_0X_1 - Z_1X_0 & 0 \\ Z_0 & Z_1 & X_0Y_1 - X_1Y_0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

3D Homogeneous Transformation

Transformation matrices will always be of the form

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} r_{xx} & r_{yx} & r_{zx} & t_x \\ r_{xy} & r_{yy} & r_{zy} & t_y \\ r_{xz} & r_{yz} & r_{zz} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

OpenGL Transformation Matrices

- You can set the matrices explicitly in OpenGL
 - `glLoadMatrixd(double mat[16])`
 - `glMultMatrixd(double mat[16])`
- Setting the matrix may be convenient when you calculate vectors instead of angles
- Killer fact:
 - The matrix is stored in COLUMN major order

Useful Vector Results

- $\mathbf{c} = \mathbf{a} \times \mathbf{b}$
 - \mathbf{c} is perpendicular to the plane formed by \mathbf{a} and \mathbf{b}
 - \mathbf{c} is zero when \mathbf{a} and \mathbf{b} are parallel
- $\mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{a})$
 - \mathbf{c} is perpendicular to \mathbf{a}
 - \mathbf{c} is in the plane formed by \mathbf{a} and \mathbf{b}
 - \mathbf{c} is zero when \mathbf{a} and \mathbf{b} are parallel