

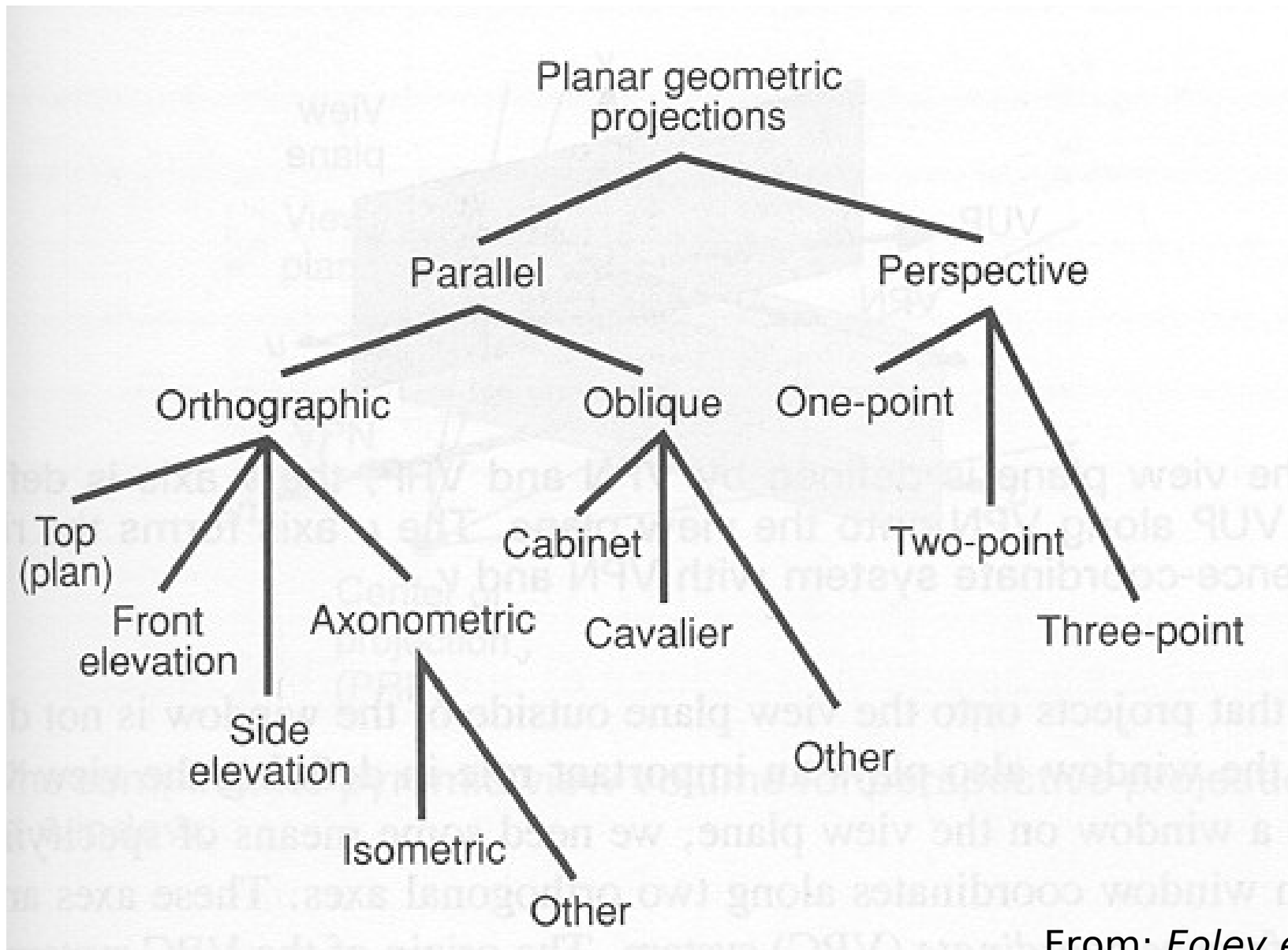
# **Drawing in 3D Projections**

**CSCI 4229/5229  
Computer Graphics  
Fall 2020**

# Types of Projections

- Parallel Projections
  - Orthogonal, isometric, ...
  - Size does not diminish with distance
- Perspective
  - Realistic based on an observer's point of view
  - Nearer bigger, farther smaller
  - One or more vanishing points

# Taxonomy of Projections

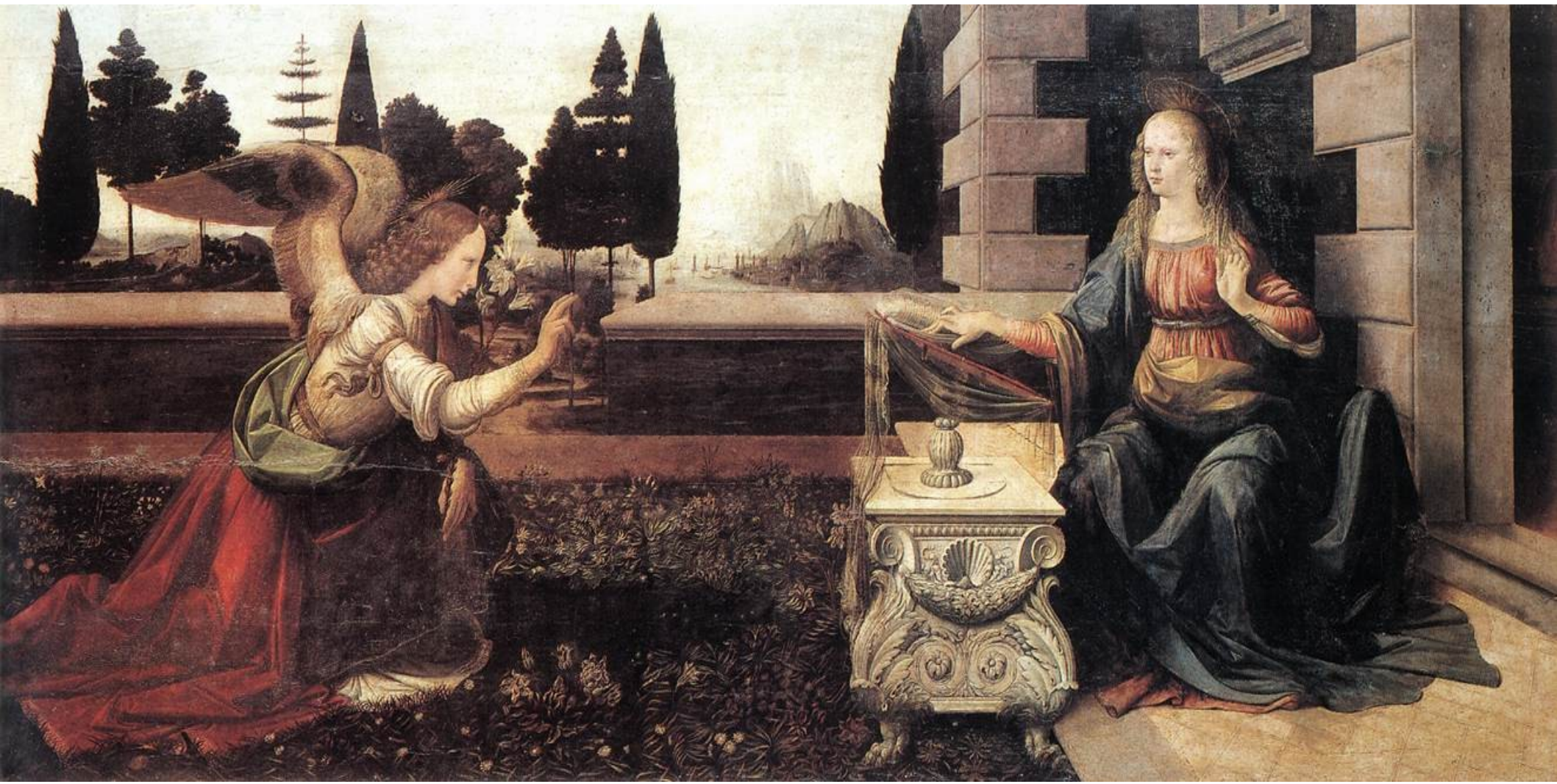


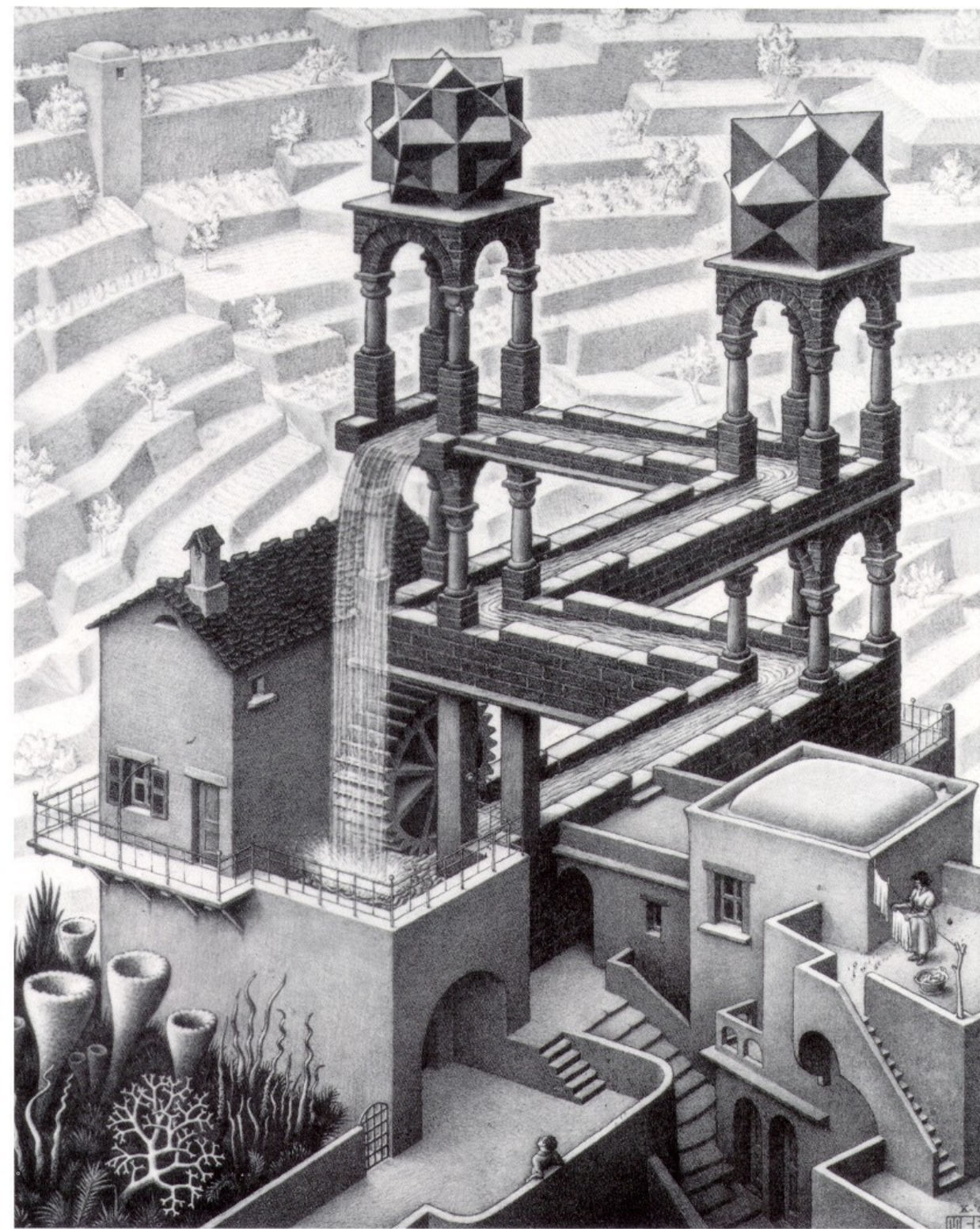
From: *Foley et al*

# Egyptian Tomb Painting



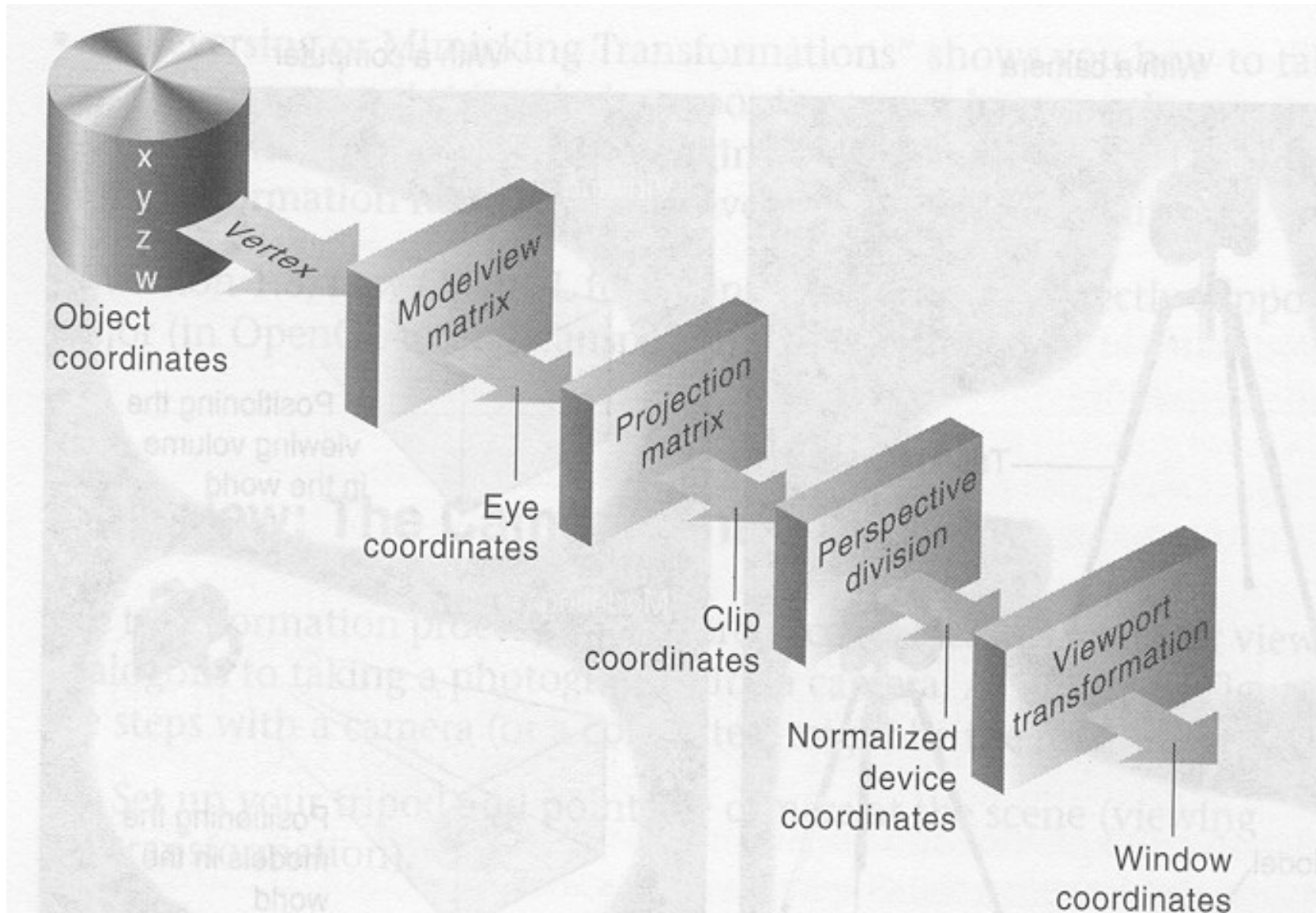
# *Annunciation* Leonardo da Vinci (1472)





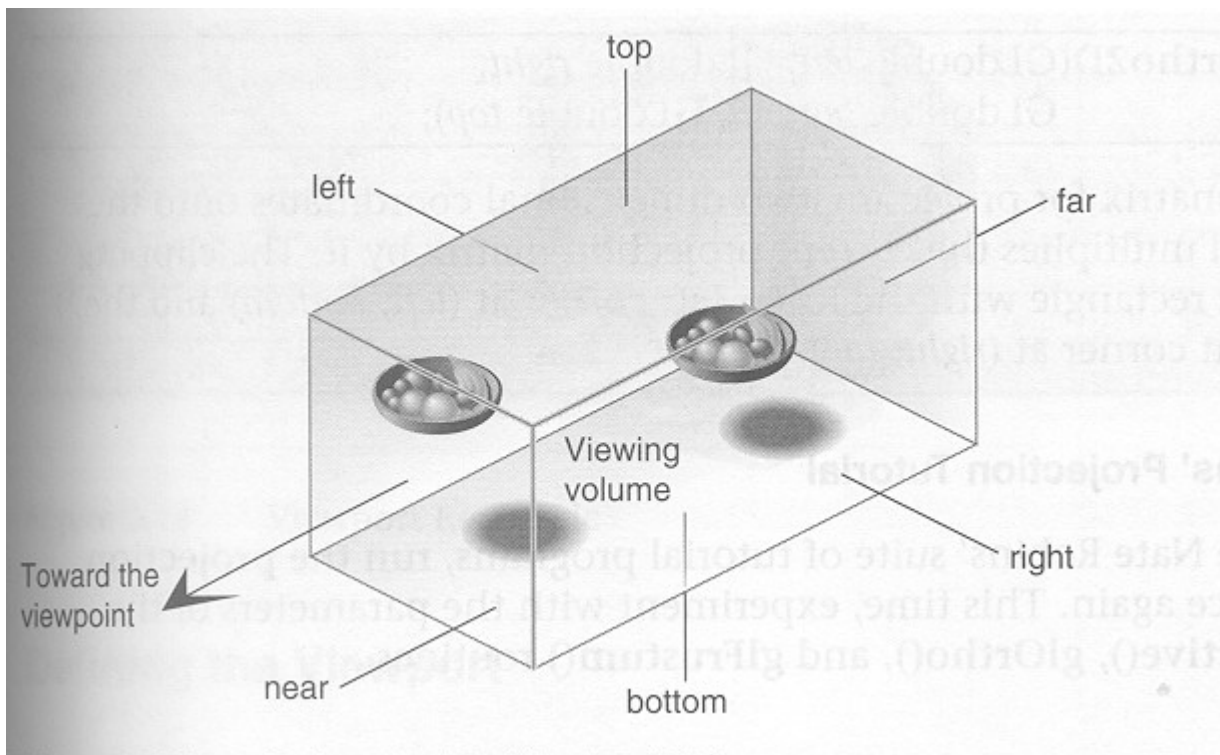
Waterfall  
M.C.  
Escher  
(1961)

# OpenGL Transformation Pipeline



# Parallel Projection

- Apply rotation matrix to map direction of projection to  $Z$  axis and up to  $Y$  axis
- Scale to canonical volume



From: *OpenGL  
Red Book*



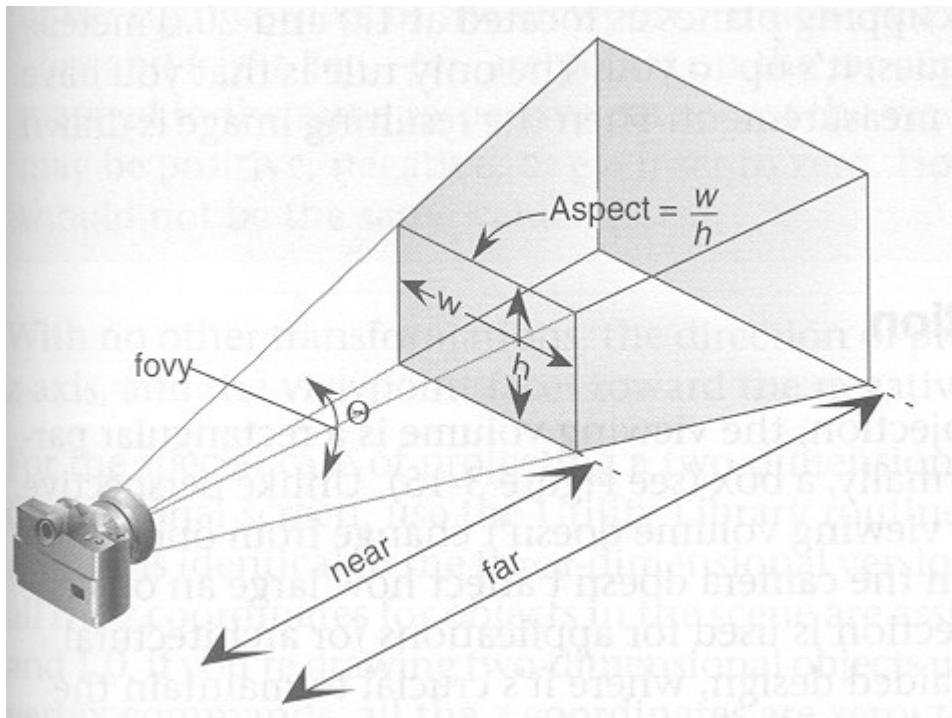
$\text{glOrtho}(x_{min}, x_{max}, y_{min}, y_{max}, z_{min}, z_{max})$

glOrtho Projection Matrix

$$\begin{pmatrix} \frac{2}{x_{max} - x_{min}} & 0 & 0 & -\frac{x_{max} + x_{min}}{x_{max} - x_{min}} \\ 0 & \frac{2}{y_{max} - y_{min}} & 0 & -\frac{y_{max} + y_{min}}{y_{max} - y_{min}} \\ 0 & 0 & \frac{-2}{z_{max} - z_{min}} & \frac{z_{max} + z_{min}}{z_{max} - z_{min}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

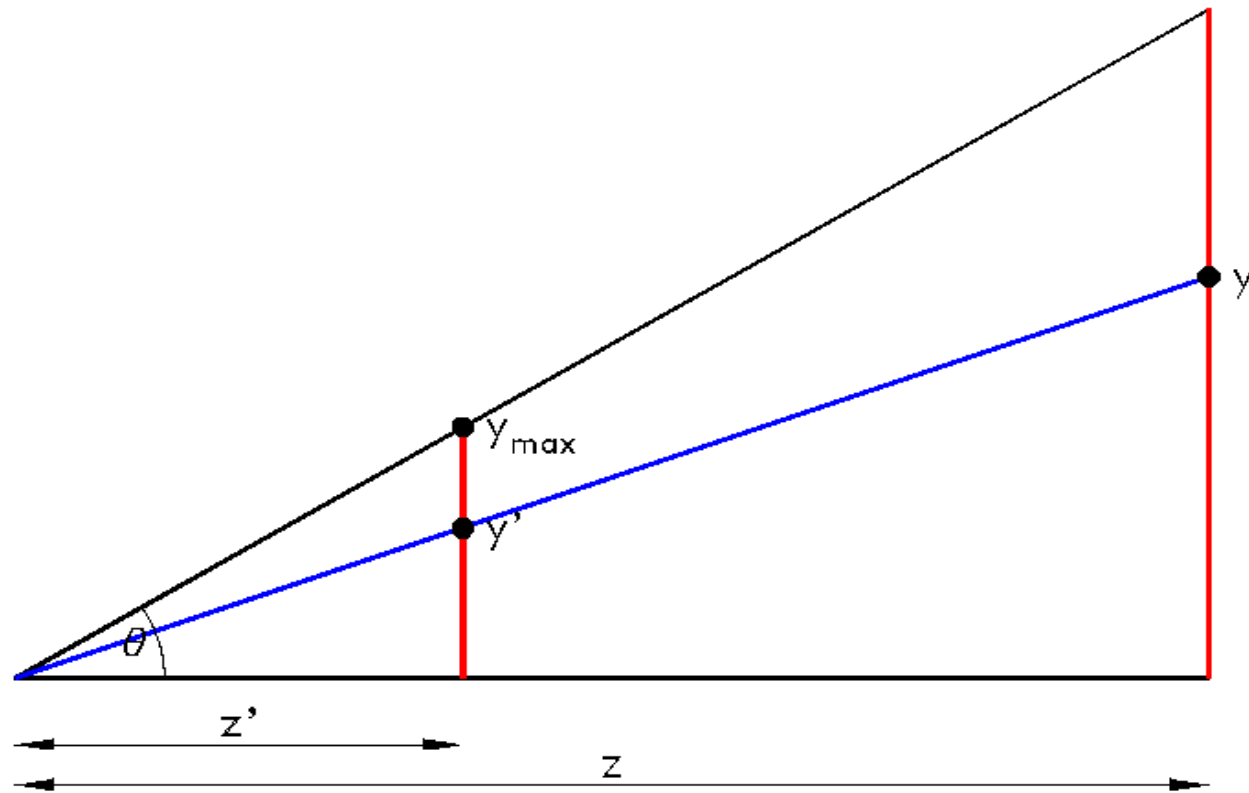
# Perspective Transformation

- Apply rotation matrix to map eye position to center of scene to negative  $Z$  and up to  $Y$  axes
- Scale  $(x,y)$  inversely proportional to distance
- Scale to canonical volume



From: *OpenGL  
Red Book*

# Perspective Transformation



Similar triangles:  $y'/z' = y/z$ , so  $y' = y/z z'$

Let  $y_{max} = 1$  (NDC),  $\tan \theta = y_{max} / z'$ ,  $z' = \cot \theta$

$$\mathbf{y' = \cot \theta y/z}$$

# Homogeneous Perspective Multiply

$$\begin{pmatrix} \frac{\cot \theta}{\text{aspect}} & 0 & 0 & 0 \\ 0 & \cot \theta & 0 & 0 \\ 0 & 0 & \frac{z_{\text{far}} + z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} & \frac{2z_{\text{far}}z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cot \theta}{\text{aspect}} x \\ \cot \theta y \\ \frac{z_{\text{far}} + z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} z + \frac{2z_{\text{far}}z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} \\ -z \end{pmatrix}$$

$$\equiv \begin{pmatrix} \frac{\cot \theta}{\text{aspect}} \frac{x}{z} \\ \cot \theta \frac{y}{z} \\ \frac{-2z_{\text{far}}z_{\text{near}}}{z(z_{\text{far}} - z_{\text{near}})} - \frac{z_{\text{far}} + z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} \end{pmatrix}$$

# gluPerspective(fovy, aspect, Znear, Zfar)

- *fovy* is the angle in the up/down direction
- *aspect* is the horizontal to vertical ratio
- *Znear* is the distance to the near clipping plane
  - Killer fact  $Znear > 0$
- *Zfar* is the distance to the far clipping plane
  - $Zfar > Znear$
- *Zfar-Znear* determines *Z* resolution since the *Z* buffer has finite precision
  - $\log_2( Zfar/Znear )$  bits lost

# gluPerspective(fovy, aspect, Znear, Zfar)

Let  $\theta = \text{fovy}/2$

gluPerspective Projection Matrix

$$\begin{pmatrix} \frac{\cot \theta}{\text{aspect}} & 0 & 0 & 0 \\ 0 & \cot \theta & 0 & 0 \\ 0 & 0 & -\frac{z_{\text{far}} + z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} & \frac{2z_{\text{far}}z_{\text{near}}}{z_{\text{far}} - z_{\text{near}}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$\text{gluLookAt}(E_x, E_y, E_z, C_x, C_y, C_z, U_x, U_y, U_z)$

- $(E_x, E_y, E_z)$  is the eye position
- $(C_x, C_y, C_z)$  is the position you look at
- $(U_x, U_y, U_z)$  is the up direction
- $C-E$  determines the distance in the  $Z$  direction
- The  $Z$  distance to each object (from  $E$ ) determines the reduction in the  $(x, y)$  direction

`gluLookAt( $E_x, E_y, E_z, C_x, C_y, C_z, U_x, U_y, U_z$ )`

Forward       $\mathbf{F} = \mathbf{C} - \mathbf{E}$

Sideways     $\mathbf{S} = \mathbf{F} \times \mathbf{U}$

Up             $\mathbf{U} = \mathbf{S} \times \mathbf{F}$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} S_x & U_x & -F_x & 0 \\ S_y & U_y & -F_y & 0 \\ S_z & U_z & -F_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -E_x \\ 0 & 1 & 0 & -E_y \\ 0 & 0 & 1 & -E_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$
$$= \begin{pmatrix} S_x & U_x & -F_x & -E_x S_x - E_y U_x + E_z F_x \\ S_y & U_y & -F_y & -E_x S_y - E_y U_y + E_z F_y \\ S_z & U_z & -F_z & -E_x S_z - E_y U_z + E_z F_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

( $\mathbf{F}$  and  $\mathbf{U}$  must be normalized)



# First Person Navigation

- Decide where you are  $(E_x, E_y, E_z)$
- Decide which way you are looking
  - $(C_x, C_y, C_z) = (E_x, E_y, E_z) + (d_x, d_y, d_z)$
- Decide up, e.g.  $(0, 0, 1)$
- Move forward to new position
  - $(E_x, E_y, E_z) += dt * (d_x, d_y, d_z)$
- Turn left and right using angle
  - $(d_x, d_y, d_z) = (\cos\theta, \sin\theta, 0)$