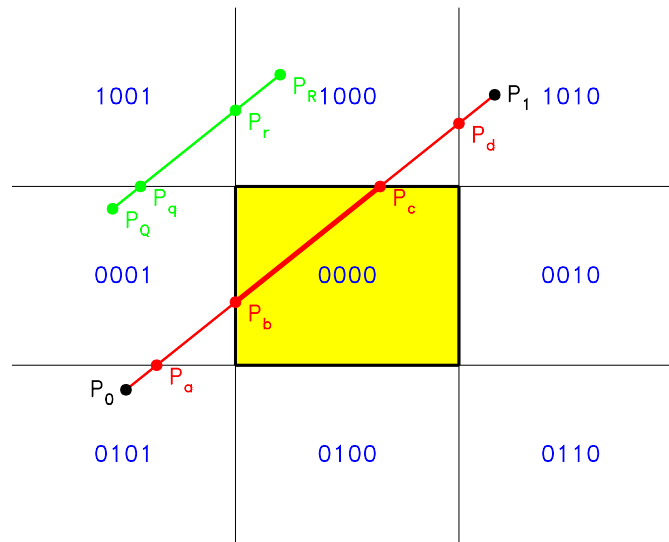


## 2D Clipping Algorithms

CSCI 4229/5229 Fall 2020

### Cohen-Sutherland



Define the clip code for a point as a four bit value TBRL, where

$$T = \begin{cases} 1 & \text{if } y > y_{max} \\ 0 & \text{if } y \leq y_{max} \end{cases} \quad B = \begin{cases} 1 & \text{if } y < y_{min} \\ 0 & \text{if } y \geq y_{min} \end{cases}$$

$$R = \begin{cases} 1 & \text{if } x > x_{max} \\ 0 & \text{if } x \leq x_{max} \end{cases} \quad L = \begin{cases} 1 & \text{if } x < x_{min} \\ 0 & \text{if } x \geq x_{min} \end{cases}$$

Let  $C_0$  and  $C_1$  be the TBRL clip code for  $P_0$  and  $P_1$ , respectively. Then

- 1: If  $C_0 = 0$  and  $C_1 = 0$  the line is completely inside the domain;
- 2: If  $C_0$  and  $C_1$  have one or more bits in common, the line is completely outside the domain;
- 3: If any of the bits in  $C_0$  or  $C_1$  are set, clip to the corresponding edge as follows, updating the end points and clip codes as appropriate

$$T: x = x_0 + (y_{max} - y_0) \frac{x_1 - x_0}{y_1 - y_0};$$

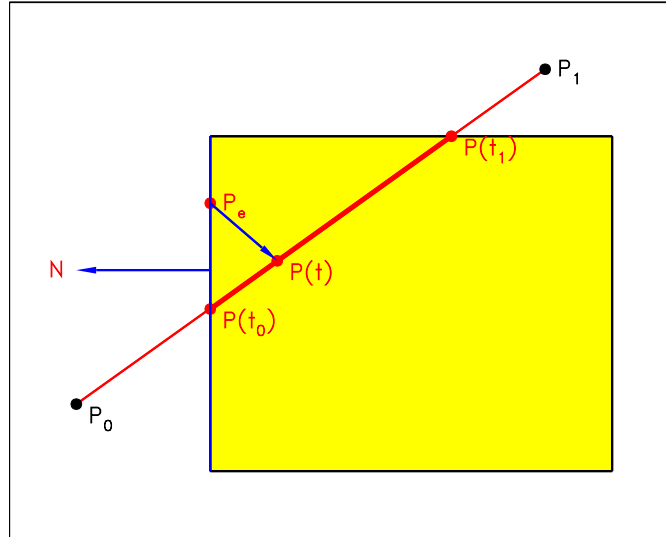
$$B: x = x_0 + (y_{min} - y_0) \frac{x_1 - x_0}{y_1 - y_0};$$

$$R: y = y_0 + (x_{max} - x_0) \frac{y_1 - y_0}{x_1 - x_0};$$

$$L: y = y_0 + (x_{min} - x_0) \frac{y_1 - y_0}{x_1 - x_0};$$

Note that after clipping the line may still miss the domain as in the case of line  $\overline{P_Q P_R}$ .

## Liang-Barsky



Let

$$\mathbf{P}(t) = (1 - t)\mathbf{P}_0 + t\mathbf{P}_1$$

For  $P(t)$  on the same edge as  $P_e$  and hence perpendicular to  $N$ ,

$$\begin{aligned} 0 &= \mathbf{N} \cdot [\mathbf{P}(t) - \mathbf{P}_e] \\ &= \mathbf{N} \cdot [(1 - t)\mathbf{P}_0 + t\mathbf{P}_1 - \mathbf{P}_e] \\ &= \mathbf{N} \cdot \mathbf{P}_0 - t\mathbf{N} \cdot \mathbf{P}_0 + t\mathbf{N} \cdot \mathbf{P}_1 - \mathbf{N} \cdot \mathbf{P}_e \end{aligned}$$

Bring all terms containing  $t$  to the left

$$t\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_1] = \mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_e]$$

Rearrange

$$t = \frac{\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_e]}{\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_1]}$$

For the left edge  $\mathbf{N} = (-1, 0)$  and  $\mathbf{P}_e = (x_{min}, y)$  so that

$$t = \frac{(-1, 0) \cdot (x_0 - x_{min}, y_0 - y)}{(-1, 0) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{x_{min} - x_0}{x_1 - x_0}$$

For the right edge  $\mathbf{N} = (1, 0)$  and  $\mathbf{P}_e = (x_{max}, y)$  so that

$$t = \frac{(1, 0) \cdot (x_0 - x_{max}, y_0 - y)}{(1, 0) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{x_0 - x_{max}}{x_0 - x_1}$$

For the bottom edge  $\mathbf{N} = (0, -1)$  and  $\mathbf{P}_e = (x, y_{min})$  so that

$$t = \frac{(0, -1) \cdot (x_0 - x, y_0 - y_{min})}{(0, -1) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{y_{min} - y_0}{y_1 - y_0}$$

For the top edge  $\mathbf{N} = (0, 1)$  and  $\mathbf{P}_e = (x, y_{max})$  so that

$$t = \frac{(0, 1) \cdot (x_0 - x, y_0 - y_{max})}{(0, 1) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{y_0 - y_{max}}{y_0 - y_1}$$