Drawing in 2D

CSCI 4229/5229
Computer Graphics
Fall 2025

Coordinate Systems

- Cartesian coordinates
  - Most commonly used
  - Left or right handed
  - 2D is a trivial case in 3D
- Polar coordinates
  - Convenient in some instances
- Curvilinear Coordinates
  - Specialized applications

2D Cartesian Coordinate Systems

- World Coordinates
  - *Xmin – Xmax* x *Ymin - Ymax*
- Normalized Device Coordinates
  - 0-1 x 0-1 or 0-1 x 0-*r* or -1 to +1
  - may be isometric
  - Viewport *Umin – Umax* x *Vmin – Vmax*
- Device coordinates
  - pixels, plotter increments
  - origin may be top-left

Transformations

- World to Normalized Device Coordinates

$$u = (x-Xmin)/(Xmax-Xmin)*(Umax-Umin) + Umin$$

$$v = (y-Ymin)/(Ymax-Ymin)*(Vmax-Vmin) + Vmin$$

- Normalized Device to World Coordinates

$$x = (u-Umin)/(Umax-Umin)*(Xmax-Xmin) + Xmin$$

$$y = (v-Vmin)/(Vmax-Vmin)*(Ymax-Ymin) + Ymin$$

- (x,y) may be outside $(Xmin–Xmax, Ymin-Ymax)$

Vector Lines

- Line from (x_0,y_0) to (x_1,y_1)
- Explicit
  - $y = (x-x_0)*(y_1-y_0)/(x_1-x_0) + y_0$
  - $x = (y-y_0)*(x_1-x_0)/(y_1-y_0) + x_0$
- Parameteric
  - $x = (1-f\,)x_0 + f\,x_1$
  - $y = (1-f\,)y_0 + f\,y_1$
  - $f = 0 \;\Rightarrow\; (x_0,y_0)$; $f = 1 \;\Rightarrow\; (x_1,y_1)$

Vector Clipping

- Cohen-Sutherland Line Clipping
  - Determine region of start and end
  - Accept, reject or clip
- Parametric Line-Clipping Algorithm
  - Calculate parameter *t*
  - $0 < t < 1$ requires clipping
- Sutherland-Hodgman Polygon Clipping
  - Clips edges of polygon
  - Successive clips to half planes
- ***OpenGL does this for you***

Cohen-Sutherland Line Clipping

- Set bits to identify outside zones
- Trivial accept or reject
- Clip non-trivial cases
- Accept or reject

Parametric Line Clipping

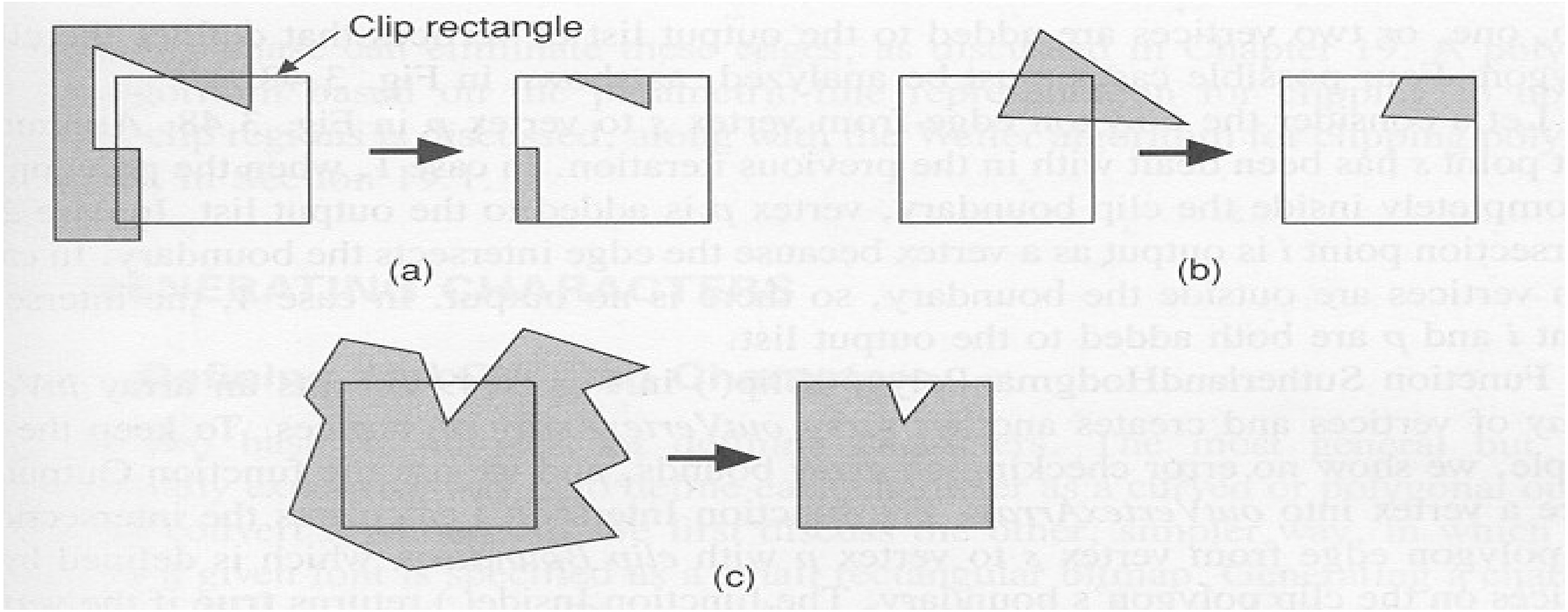
- Cohen-Sutherland may require up to 4 clips
- Parameteric algorithm more efficient
  - Original Cyrus-Beck
  - Improved by Liang-Barsky
- Readily extends to 3D and irregular windows
- Basic equation for line from P_0 to P_1

$$t = (N \cdot [P_0 - P_E]) \,/\, (N \cdot [P_0 - P_1])$$

N is the outside normal

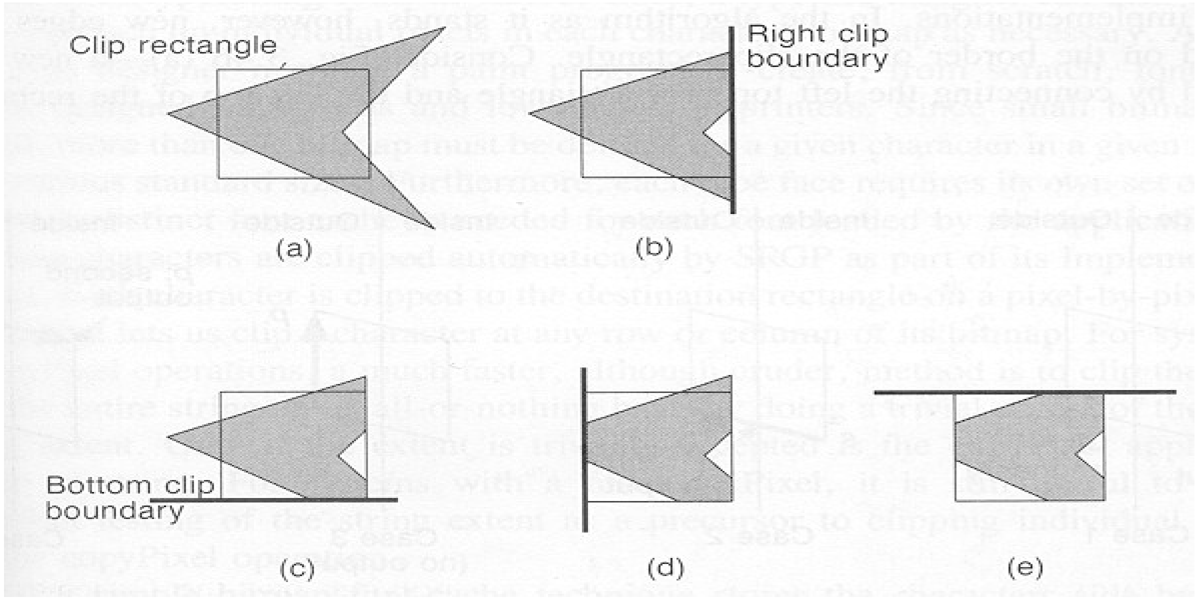
P_E is on the edge

Polygon Clipping Challenges



From *Foley,van Dam Feiner & Hughes*

Polygon Clipping Algorithm



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