

Parametric Surfaces

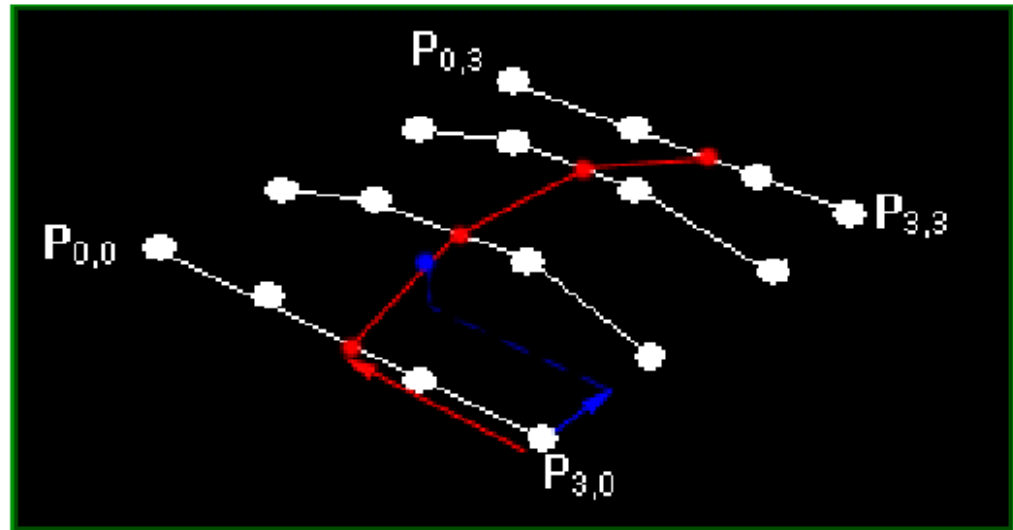
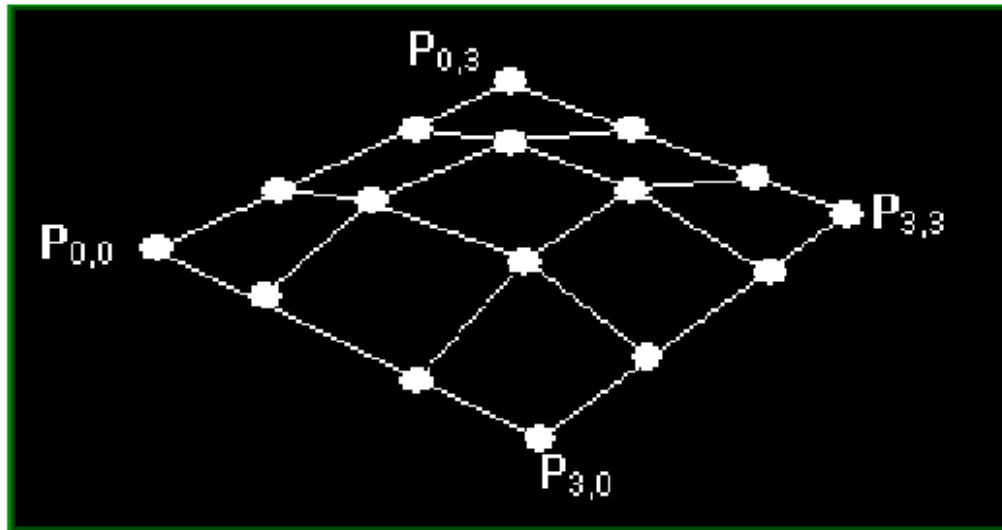
**CSCI 4229/5229
Computer Graphics
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Bézier Surfaces

- In one dimension
 - $C_n(t) = \sum_{i=0}^n B_i^n(t) P_i, \quad t \in [0,1]$
- In two dimensions
 - $S_{n,m}(t,r) = \sum_{i=0}^n B_i^n(t) \sum_{j=0}^m B_j^m(r) P_{ij}, \quad t,r \in [0,1]$
- P_{ij} are points in 3D or 4D
- Convex linear combination of points P_{ij}
 - Entire curve is in convex hull of points
 - Surface passes through 4 corner points
- Curve is smooth and differentiable

2D Cubic Bézier Surface

- 16 Control points
- Corner points set surface
- Interior points stretches surface



Bicubic Bézier Patch

$$\begin{aligned}
 P &= (1-v)^3 \left((1-u)^3 P_{00} + 3(1-u)^2 u P_{01} + 3(1-u) u^2 P_{02} + u^3 P_{03} \right) \\
 &+ 3(1-v)^2 v \left((1-u)^3 P_{10} + 3(1-u)^2 u P_{11} + 3(1-u) u^2 P_{12} + u^3 P_{13} \right) \\
 &+ 3(1-v) v^2 \left((1-u)^3 P_{20} + 3(1-u)^2 u P_{21} + 3(1-u) u^2 P_{22} + u^3 P_{23} \right) \\
 &+ v^3 \left((1-u)^3 P_{30} + 3(1-u)^2 u P_{31} + 3(1-u) u^2 P_{32} + u^3 P_{33} \right) \\
 &= (1-u)^3 \left((1-v)^3 P_{00} + 3(1-v)^2 v P_{10} + 3(1-v) v^2 P_{20} + v^3 P_{30} \right) \\
 &+ 3(1-u)^2 u \left((1-v)^3 P_{01} + 3(1-v)^2 v P_{11} + 3(1-v) v^2 P_{21} + v^3 P_{31} \right) \\
 &+ 3(1-u) u^2 \left((1-v)^3 P_{02} + 3(1-v)^2 v P_{12} + 3(1-v) v^2 P_{22} + v^3 P_{32} \right) \\
 &+ u^3 \left((1-v)^3 P_{03} + 3(1-v)^2 v P_{13} + 3(1-v) v^2 P_{23} + v^3 P_{33} \right)
 \end{aligned}$$

Bicubic Bézier Patch Normal

$$\begin{aligned}
 \frac{\partial P}{\partial u} &= -3(1-u)^2 \left((1-v)^3 P_{00} + 3(1-v)^2 v P_{10} + 3(1-v) v^2 P_{20} + v^3 P_{30} \right) \\
 &+ 3(1-3u)(1-u) \left((1-v)^3 P_{01} + 3(1-v)^2 v P_{11} + 3(1-v) v^2 P_{21} + v^3 P_{31} \right) \\
 &+ 3u(2-3u) \left((1-v)^3 P_{02} + 3(1-v)^2 v P_{12} + 3(1-v) v^2 P_{22} + v^3 P_{32} \right) \\
 &+ 3u^2 \left((1-v)^3 P_{03} + 3(1-v)^2 v P_{13} + 3(1-v) v^2 P_{23} + v^3 P_{33} \right) \\
 \frac{\partial P}{\partial v} &= -3(1-v)^2 \left((1-u)^3 P_{00} + 3(1-u)^2 u P_{01} + 3(1-u) u^2 P_{02} + u^3 P_{03} \right) \\
 &+ 3(1-3v)(1-v) \left((1-u)^3 P_{10} + 3(1-u)^2 u P_{11} + 3(1-u) u^2 P_{12} + u^3 P_{13} \right) \\
 &+ 3v(2-3v) \left((1-u)^3 P_{20} + 3(1-u)^2 u P_{21} + 3(1-u) u^2 P_{22} + u^3 P_{23} \right) \\
 &+ 3v^2 \left((1-u)^3 P_{30} + 3(1-u)^2 u P_{31} + 3(1-u) u^2 P_{32} + u^3 P_{33} \right)
 \end{aligned}$$

$$N = \frac{\partial P}{\partial u} \times \frac{\partial P}{\partial v}$$

Surfaces in OpenGL

- Two-dimensional Evaluators
- Can be used to generate vertexes, normals, colors and textures
- Curve defined analytically using Bezier surfaces
- Evaluated at discrete points and rendered using polygons

Surfaces in OpenGL

- `glEnable()`
 - Enables types of data to generate
 - `GL_AUTO_NORMAL` generates normals for you
- `glMap2d()`
 - Defines control points and domain
- `glEvalCoord2d()`
 - Generates a data point
- `glMapGrid2d()` & `glEvalMesh2()`
 - Generates a series of data points

`glMap2d(type, Umin, Umax, Ustride, Uorder,
Vmin, Vmax, Vstride, Vorder, points)`

- *type* of data to generate
 - `GL_MAP1_VERTEX_3[4]`
 - `GL_MAP1_NORMAL`
 - `GL_MAP1_COLOR_4`
 - `GL_MAP1_TEXTURE_COORD_[1-4]`
- *Umin&Umax* and *Vmin&Vmax* are limits (often 0&1)
- *Ustride* is the number of values in data (3,4)
- *Vstride* is the number of values in a row of data
- *Uorder* & *Vorder* is the order of the curve (4=cubic)
- *points* is the array of data points (16 for bi-cubic)
- **Remember to also call `glEnable()`**

glEvalCoord2d(u,v)

- Generate one vertex for each glMap2d() type currently active (e.g. texture, normal, vertex)
- To generate the whole surface, loop over quads and call glEvalCoord2d() once for each vertex
- Exercise entire parameter space
 - u from Umin to Umax (0 to 1)
 - v from Vmin to Vmax (0 to 1)

Generating a complete surface

- `glMapGrid2d(N , U1 , U2 , M , V1 , V2)`
- `glEvalMesh2(mode , N1 , N2 , M1 , M2)`
- This is equivalent to

```
for (j=M1;j<M2;j++)
{
    glBegin(GL_QUAD_STRIP);
    for (i=N1;i<=N2;i++)
    {
        glEvalCoord1(U1+i*(U2-U1)/N , V1+j*(V2-V1)/M);
        glEvalCoord1(U1+i*(U2-U1)/N , V1+(j+1)*(V2-V1)/M);
    }
    glEnd();
}
```

The Utah Teapot

- Generated by Martin Newell in 1975
 - 32 Patches specified as Bezier surfaces
 - 10 Base patches with reflections
 - 126 control points
- Complex shape
 - Hole in handle
 - Hollow spout
- Non-convex
 - Can cast shadows on itself



The Utah Teapot: Then and Now

