Shadows

CSCI 4229/5229
Computer Graphics
Fall 2025

Shadows in Computer Graphics

- Shadows are important to realism
  - Depth cues
  - Relative positions of objects
- Doesn't “just happen” when lighting is turned on
  - Nor is there a glEnable(GL_SHADOWS)
- Shadows require the scene to be rendered multiple times (at least 2, typically 4)
- Recent (~2000) addition to real time graphics
  - Very compute intensive

Colorado Fall Colors

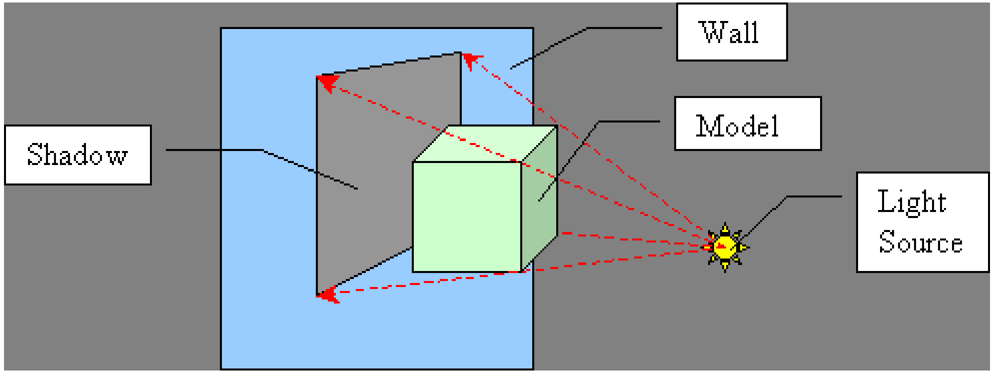
Soft vs. Hard Shadows

- Hard shadow – *point* light source
- Soft shadow – *area* light source

- Hard shadows can be done in OpenGL
- Soft shadows not practical in real time
  - Can be approximated by multiple point sources
  - Current research topic

Planar Shadows

- Projects object on surface
- Simplest shadows
- Fast but very limited
- The problem:
  - Surface defined by point *E* and normal *N*
  - L is the light
  - *P* is on the object
  - Find *P'* the projection of *P* on the surface



Extend $\vec{LP}$ to P'

$$P' = L + \lambda(P - L)$$

Let P' be in the plane

$$(P' - E) \cdot N = 0$$

Expand P' to

$$(L + \lambda(P - L) - E) \cdot N = 0$$

Then

$$\lambda = \frac{(E - L) \cdot N}{(P - L) \cdot N}$$

so that

$$P' = L + \frac{(E - L) \cdot N}{(P - L) \cdot N}(P - L)$$

Define

$$e = E \cdot N, \quad l = L \cdot N, \quad c = (E - L) \cdot N = e - l$$

Then

$$P' = L + \frac{c}{P \cdot N - l}(P - L)$$

$$P' = L + \frac{c}{P \cdot N - l}(P - L)$$

Expand x component

$$x' = L_x + \frac{cP_x - cL_x}{N_xP_x + N_yP_y + N_zP_z - l}$$
$$= \frac{N_xP_xL_x + N_yP_yL_x + N_zP_zL_x - lL_x + cP_x - cL_x}{N_xP_x + N_yP_y + N_zP_z - l}$$
$$= \frac{(N_xL_x + c)P_x + (N_yL_x)P_y + (N_zL_x)P_z - eL_x}{N_xP_x + N_yP_y + N_zP_z - l}$$

Therefore

$$x' = \frac{(N_x L_x + c)P_x + (N_y L_x)P_y + (N_z L_x)P_z - eL_x}{N_x P_x + N_y P_y + N_z P_z - l}$$
$$y' = \frac{(N_x L_y)P_x + (N_y L_y + c)P_y + (N_z L_y)P_z - eL_y}{N_x P_x + N_y P_y + N_z P_z - l}$$
$$z' = \frac{(N_x L_z)P_x + (N_y L_z)P_y + (N_z L_z + c)P_z - eL_z}{N_x P_x + N_y P_y + N_z P_z - l}$$

so that

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} L_x N_x + c & L_x N_y & L_x N_z & -eL_x \\ L_y N_x & L_y N_y + c & L_y N_z & -eL_y \\ L_z N_x & L_z N_y & L_z N_z + c & -eL_z \\ N_x & N_y & N_z & -l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$