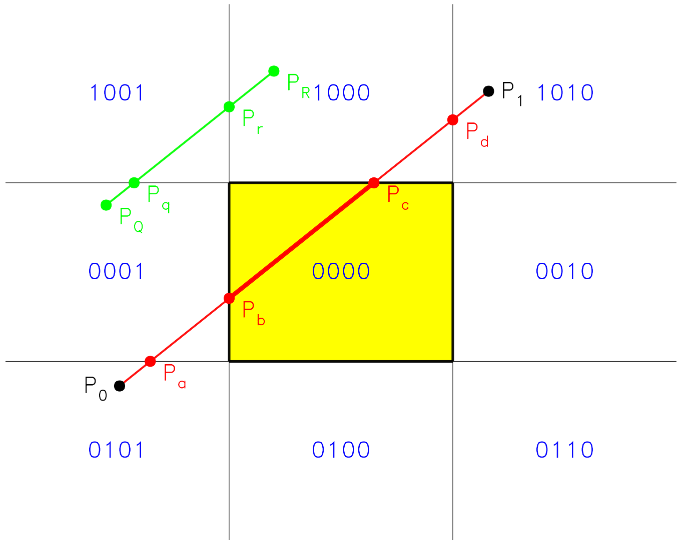
2D Clipping Algorithms

CSCI 4229/5229

Cohen-Sutherland



Define the clip code for a point as a four bit value TBRL, where

$$T = \begin{cases} 1 & \text{if } y > y_{max} \\ 0 & \text{if } y \leq y_{max} \end{cases} \qquad B = \begin{cases} 1 & \text{if } y < y_{min} \\ 0 & \text{if } y \geq y_{min} \end{cases}$$

$$R = \begin{cases} 1 & \text{if } x > x_{max} \\ 0 & \text{if } x \leq x_{max} \end{cases} \qquad L = \begin{cases} 1 & \text{if } x < x_{min} \\ 0 & \text{if } x \geq x_{min} \end{cases}$$

Let C_0 and C_1 be the TBRL clip code for P_0 and P_1, respectively. Then

1: If $C_0 = 0$ and $C_1 = 0$ the line is completely inside the domain;

2: If C_0 and C_1 have one or more bits in common, the line is completely outside the domain;

3: If any of the bits in C_0 or C_1 are set, clip to the corresponding edge as follows, updating the end points and clip codes as appropriate

T: $x = x_0 + (y_{max} - y_0)\frac{x_1 - x_0}{y_1 - y_0}$;

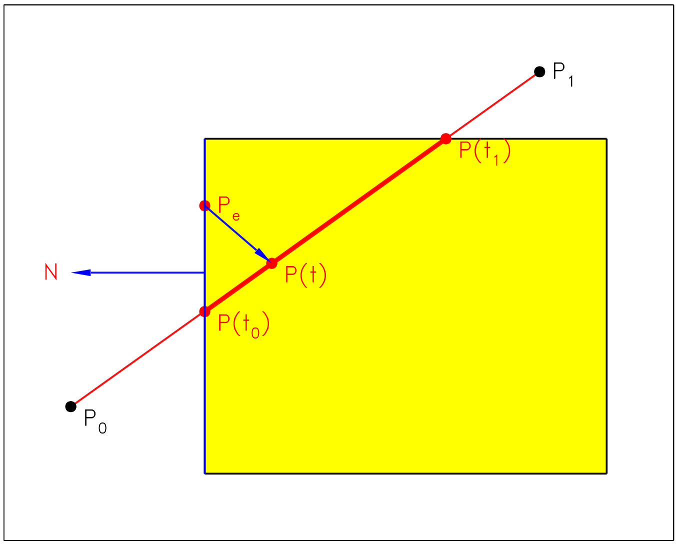
B: $x = x_0 + (y_{min} - y_0)\frac{x_1 - x_0}{y_1 - y_0}$;

R: $y = y_0 + (x_{max} - x_0)\frac{y_1 - y_0}{x_1 - x_0}$;

L: $y = y_0 + (x_{min} - x_0)\frac{y_1 - y_0}{x_1 - x_0}$;

Note that after clipping the line may still miss the domain as in the case of line $\overline{P_Q P_R}$.

Liang-Barsky



Let

$$\mathbf{P}(t) = (1-t)\mathbf{P}_0 + t\mathbf{P}_1$$

For $P(t)$ on the same edge as P_e and hence perpendicular to N,

$$\begin{aligned} 0 &= \mathbf{N} \cdot [\mathbf{P}(t) - \mathbf{P}_e] \\ &= \mathbf{N} \cdot [(1-t)\mathbf{P}_0 + t\mathbf{P}_1 - \mathbf{P}_e] \\ &= \mathbf{N} \cdot \mathbf{P}_0 - t\mathbf{N} \cdot \mathbf{P}_0 + t\mathbf{N} \cdot \mathbf{P}_1 - \mathbf{N} \cdot \mathbf{P}_e \end{aligned}$$

Bring all terms containing t to the left

$$t\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_1] = \mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_e]$$

Rearrange

$$t = \frac{\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_e]}{\mathbf{N} \cdot [\mathbf{P}_0 - \mathbf{P}_1]}$$

For the left edge $\mathbf{N} = (-1, 0)$ and $\mathbf{P}_e = (x_{min}, y)$ so that

$$t = \frac{(-1,0) \cdot (x_0 - x_{min}, y_0 - y)}{(-1,0) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{x_{min} - x_0}{x_1 - x_0}$$

For the right edge $\mathbf{N} = (1, 0)$ and $\mathbf{P}_e = (x_{max}, y)$ so that

$$t = \frac{(1,0) \cdot (x_0 - x_{max}, y_0 - y)}{(1,0) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{x_0 - x_{max}}{x_0 - x_1}$$

For the bottom edge $\mathbf{N} = (0, -1)$ and $\mathbf{P}_e = (x, y_{min})$ so that

$$t = \frac{(0,-1) \cdot (x_0 - x, y_0 - y_{min})}{(0,-1) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{y_{min} - y_0}{y_1 - y_0}$$

For the top edge $\mathbf{N} = (0, 1)$ and $\mathbf{P}_e = (x, y_{max})$ so that

$$t = \frac{(0,1) \cdot (x_0 - x, y_0 - y_{max})}{(0,1) \cdot (x_0 - x_1, y_0 - y_1)} = \frac{y_0 - y_{max}}{y_0 - y_1}$$