

# **Ray Tracing: Special Topics**

**CSCI 4239/5239**

**Advanced Computer Graphics  
Spring 2021**

# Theoretical foundations

Ray Tracing from the Ground Up Chapters 13-15

- Bidirectional Reflectance Distribution Function
  - BRDF
  - Describes how light is reflected on each bounce
  - Chains to transfer colors

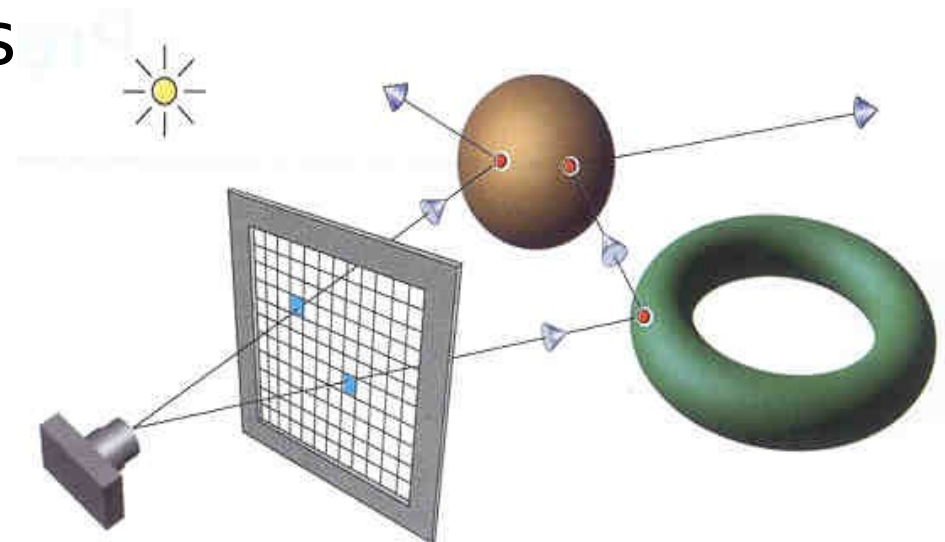


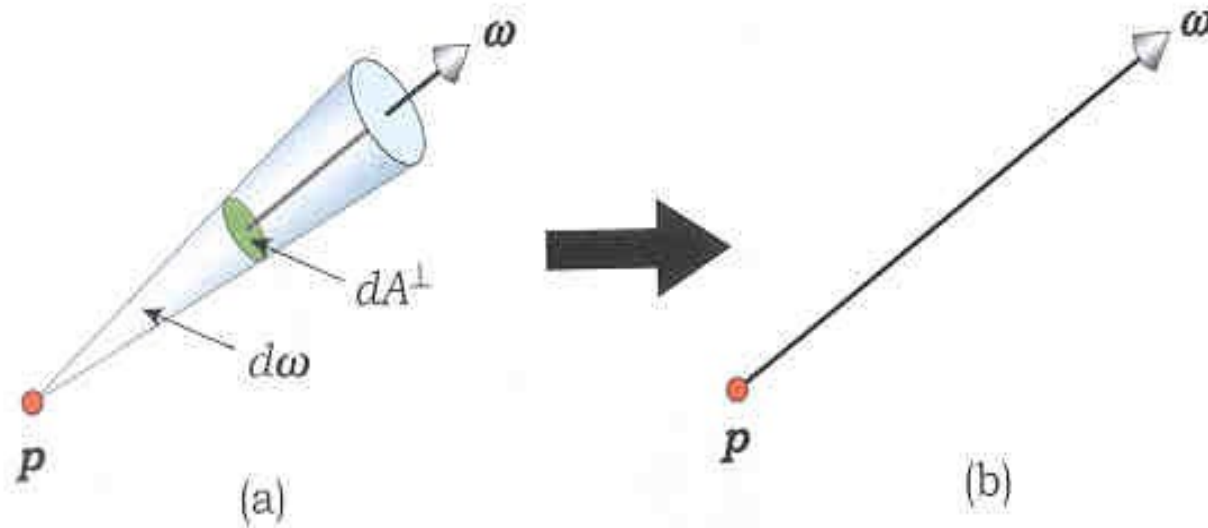
Figure 1. The ray-tracing process.

# Radiometric Quantities

- Radiant Energy  $Q$  (J)
- Radiant Flux  $\phi = dQ/dt$  (W)
- Radiant Flux Density  $d\phi/dA$  ( $W/m^2$ )
- Irradiance  $E$  [Arriving flux density]
- Radiant exitance  $M$  [Leaving flux density]
- Radiant Intensity  $I$   $d\phi/d\omega$  ( $W/m^2/sr$ )
- Radiance  $L$   $d^2\phi/dAd\omega$  ( $W/m^2/sr$ )

# Ray Properties

- Radiance is constant along rays
- Radiance can be defined at the eye
- Radiance can be defined at a point

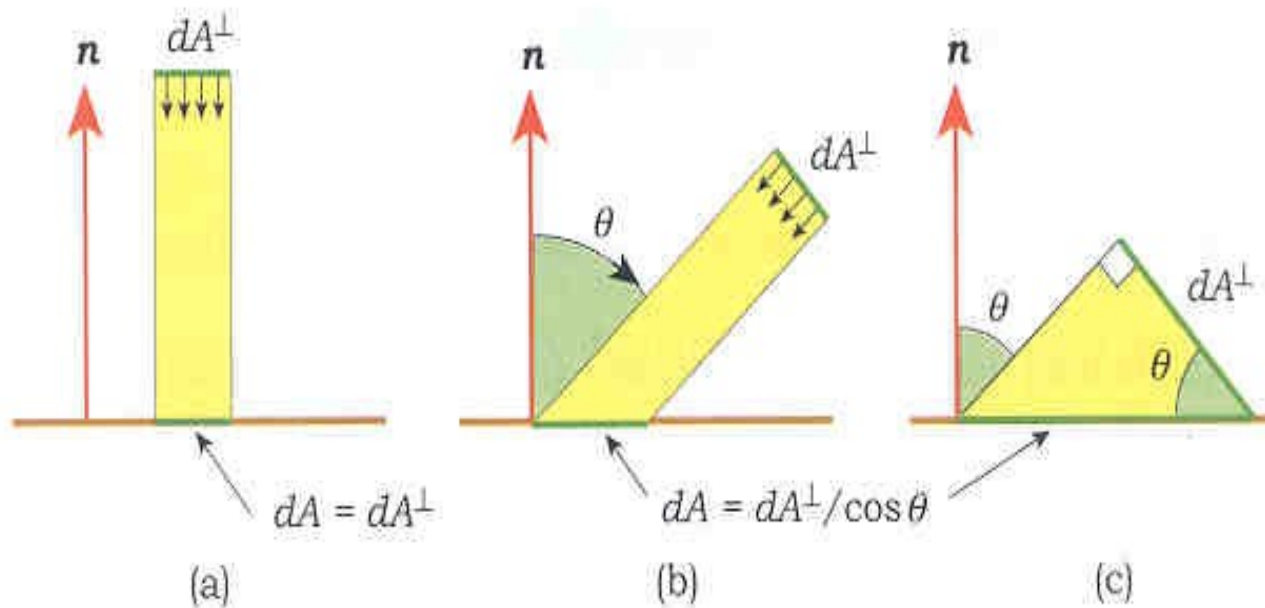


**Figure 13.1.** (a) Radiant flux in a cone of incident angles  $d\omega$  passing through a surface element  $dA^\perp$ . (b) In the limit  $d\omega \rightarrow 0$  and  $dA \rightarrow 0$ , the radiance is defined as coming from a single direction  $\omega$ . The point  $p$  can be an arbitrary point in space.

# Angular Dependence on Irradiance

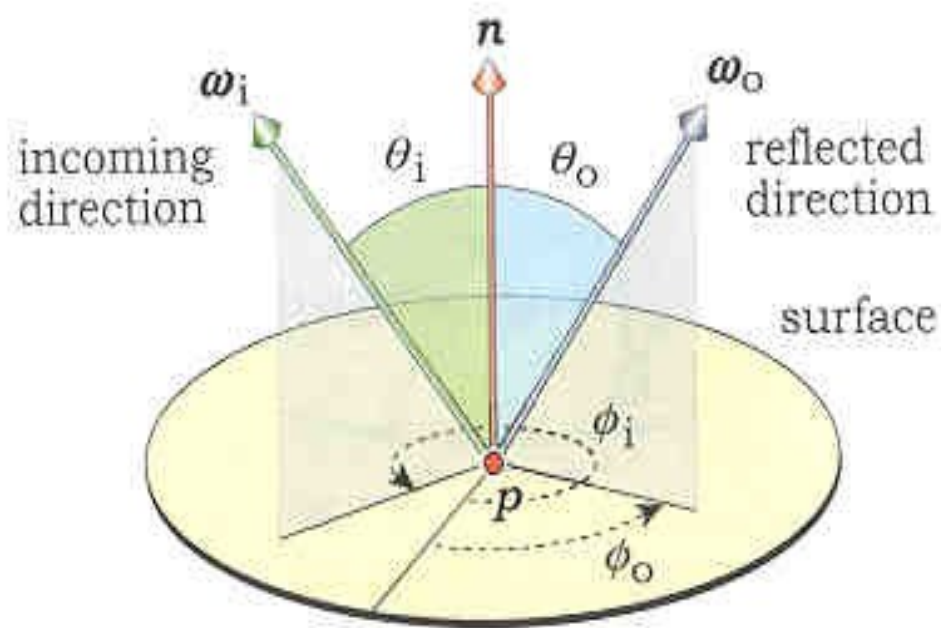
- Lambert's Law

- $L = d^2\phi/dA \cdot d\omega \cdot \cos\theta$



**Figure 13.2.** (a) and (b) Irradiance spreads out over a larger area as the incidence angle  $\theta$  increases. (c) An enlarged view of the incident beam.

# Notation and Directions



**Figure 13.3.** The incoming direction  $\omega_i$  and reflected direction  $\omega_o$  point away from the surface and are on the same side of the surface as the normal. Each direction is defined by its polar and azimuth angles  $(\theta, \phi)$ . These are arbitrary directions; for perfect mirror reflection,  $\phi_o = \phi_i \pm \pi$ , as illustrated in Figure 24.2(b).

# BRDF

- Definition

- $f(p, \omega_i, \omega_o) = dL_o(p, \omega) / dL_i(p, \omega) \cos \theta_i d\omega_i$

- Properties

- Reciprocity

- $f(p, \omega_i, \omega_o) = f(p, \omega_o, \omega_i)$

- Linearity

- Sum all BRDFs at a point

- Conservation of energy

- Total re-radiated energy must be less than incident

# Common BDRFs

- Diffuse  $f(p, \omega_i, \omega_o) = M_d(p)$

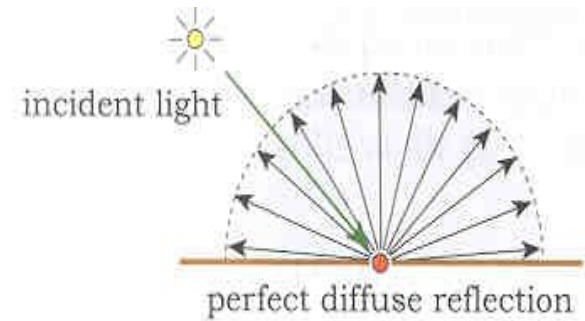


Figure 13.6. Light being scattered from a perfectly diffuse surface.

- Specular  $f(p, \omega_i, \omega_o) = M_s(p) (R \cdot \omega_o)^S$

$$- R = 2(N \cdot L)N - L$$

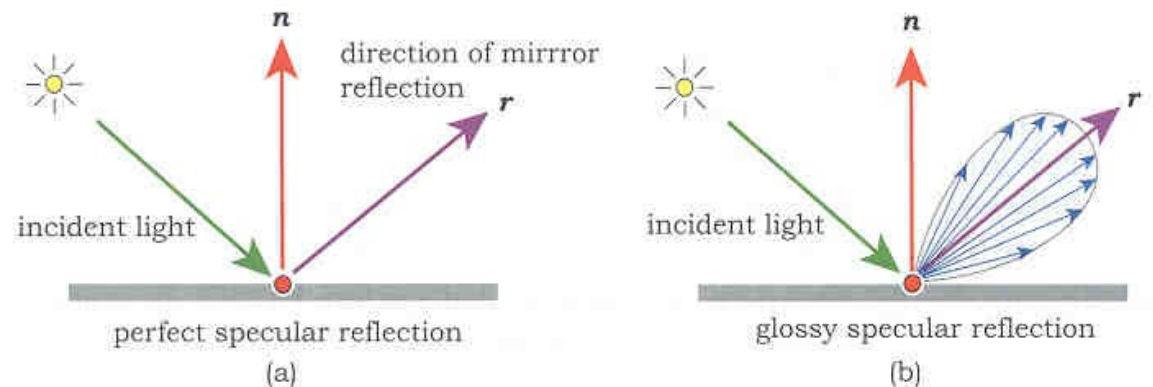
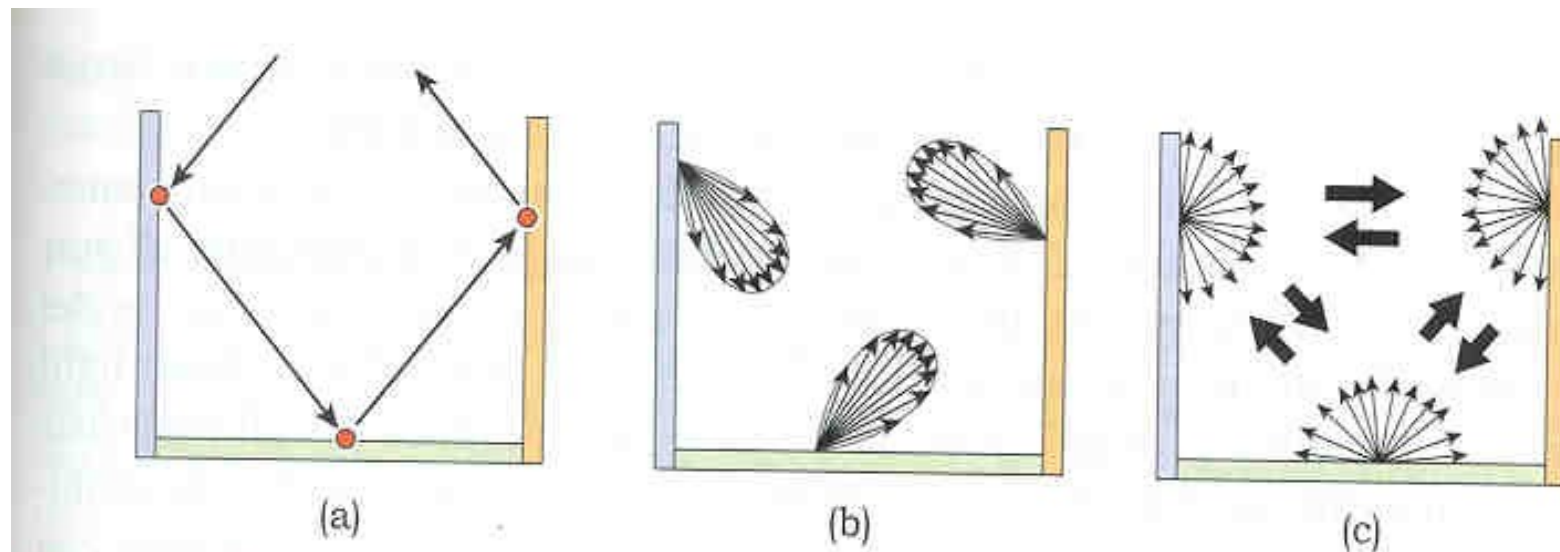


Figure 14.3. (a) Perfect specular reflection; (b) glossy specular reflection.

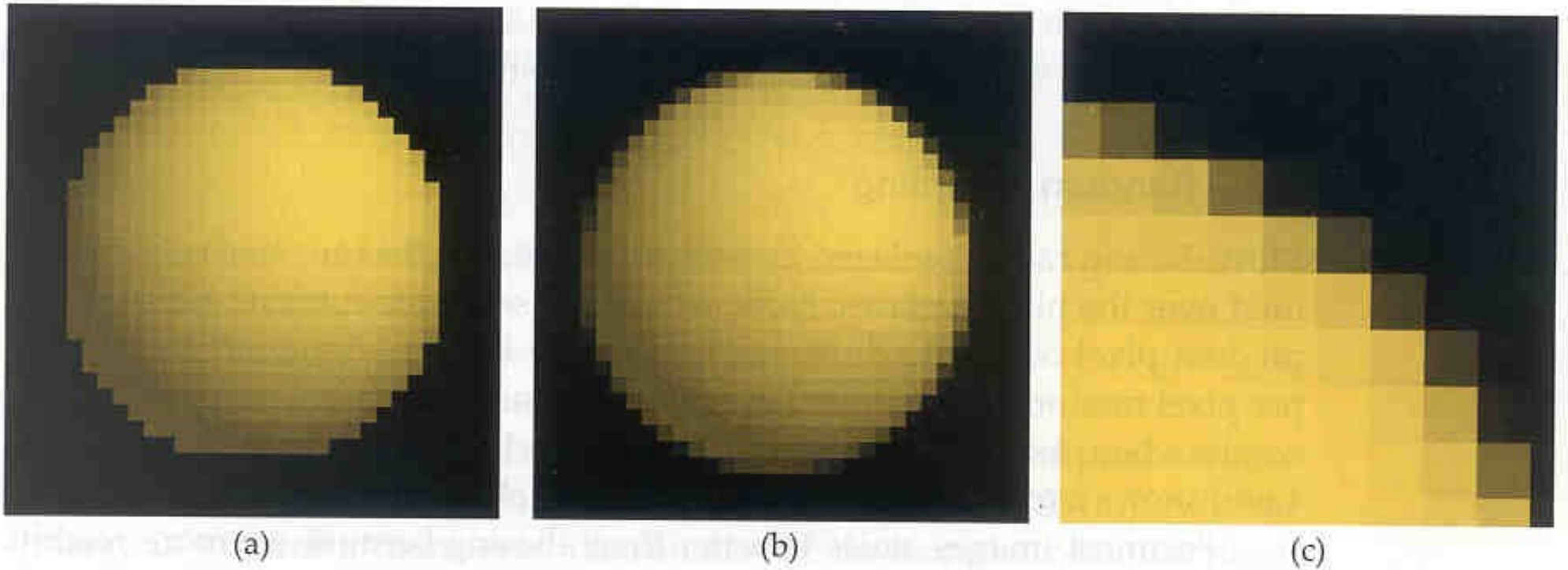


# Bouncing Rays from Surfaces



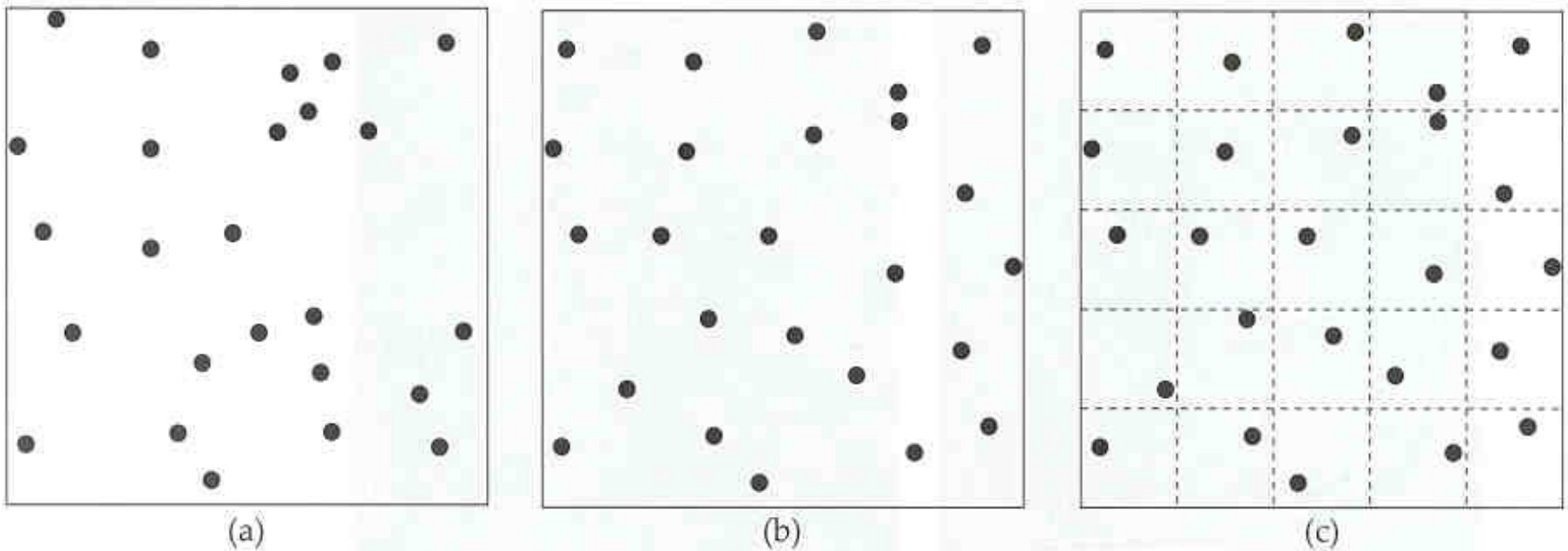
**Figure 14.4.** (a) Mirror reflection can be modeled by tracing a single reflected ray at each hit point; (b) modeling glossy specular light transport between surfaces requires many rays to be traced per pixel; (c) modeling perfect diffuse light transport between surfaces also requires many rays to be traced per pixel.

# Antialiasing



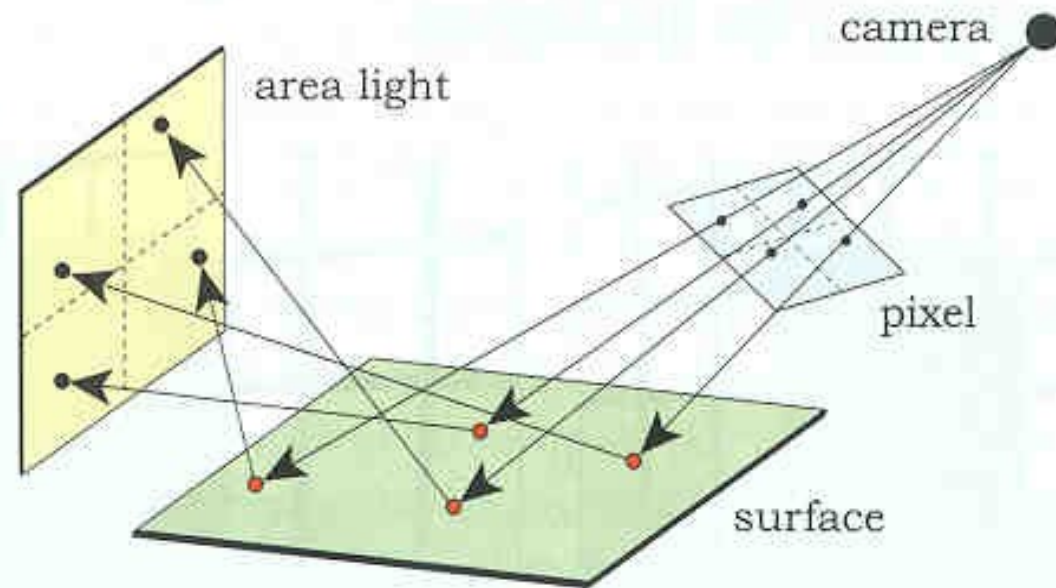
**Figure 4.4.** Shaded sphere: (a) one sample per pixel; (b) 16 samples per pixel; (c) enlarged view of top-right section of (b).

# Super-sampling Pixels



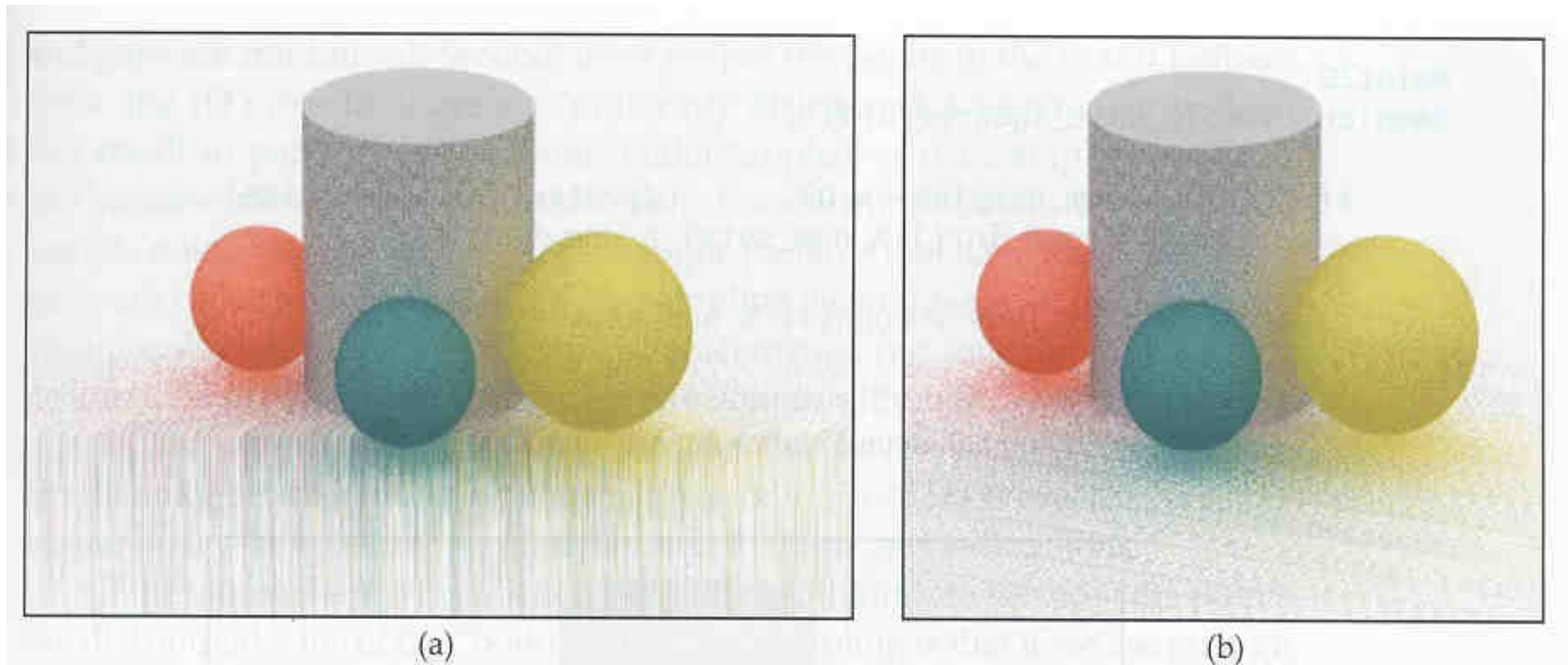
**Figure 4.7.** (a) 25 random samples in a pixel; (b) 25 jittered samples; (c) same as (b) but with sub-grid lines shown.

# Super-sampling Area Lights



**Figure 5.2.** Shading a surface with an area light and four samples per pixel.

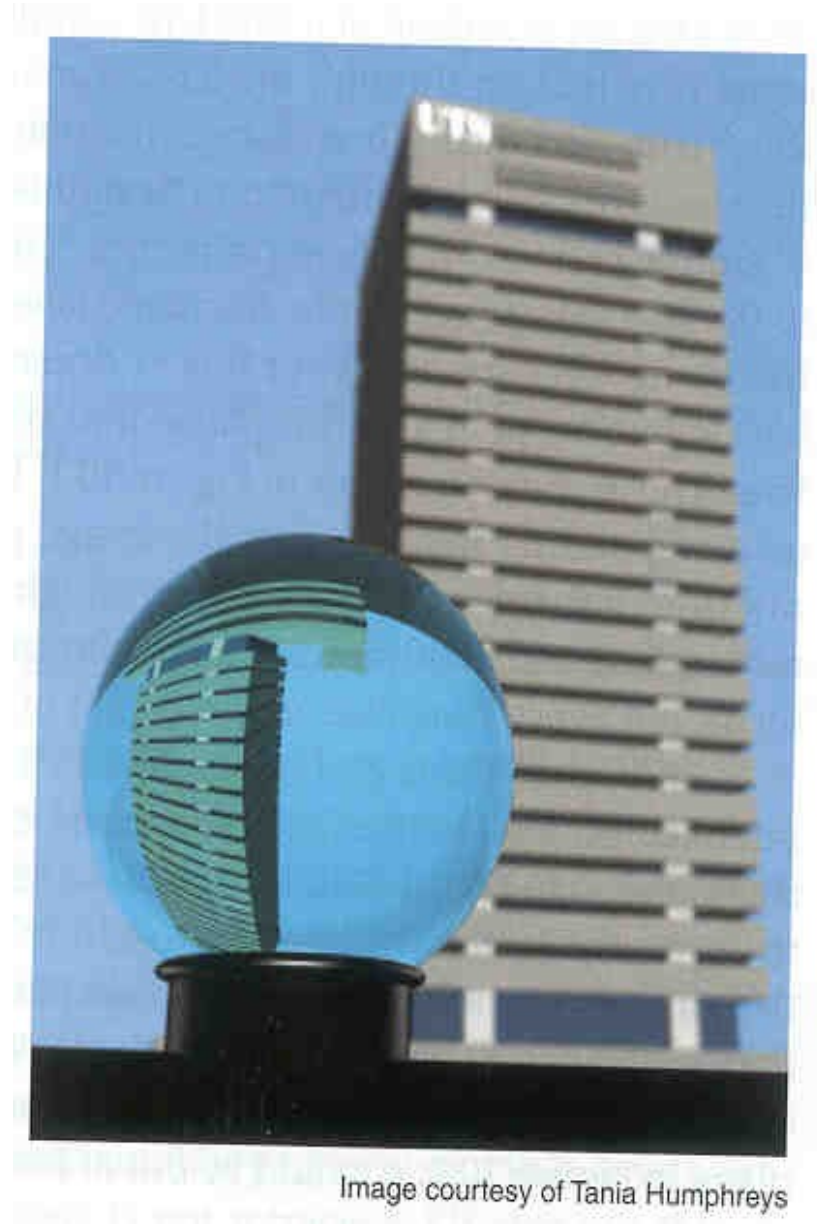
# Side-effects of Sampling Pattern



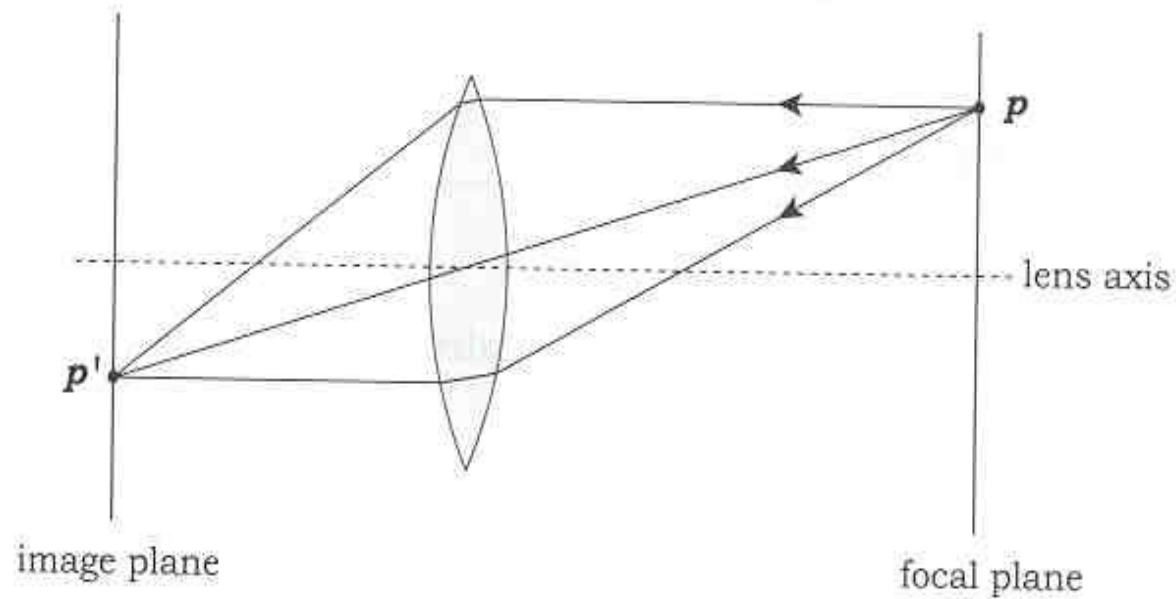
**Figure 5.6.** Global illumination images that exhibit bad aliasing caused by using the same samples in vertical columns (a) and in a regular horizontal displacement (b).

# Depth of Field

- Important for realism
  - Background is “fuzzy”
- Partly out of focus
  - Imperfect optics
  - Turbulence
- Graphic backgrounds are often too perfect

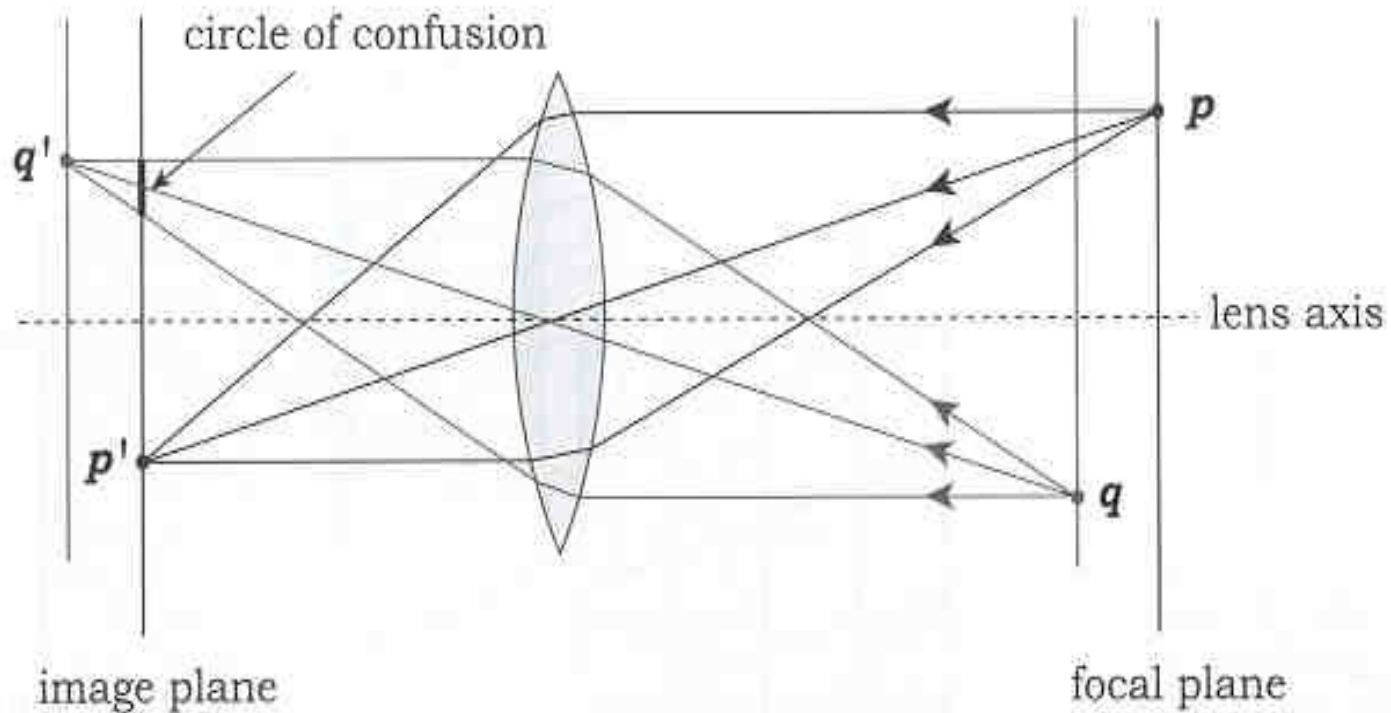


# Thin Lens Theory



**Figure 10.1.** Cross section through a thin lens showing a focal plane and its corresponding image plane.

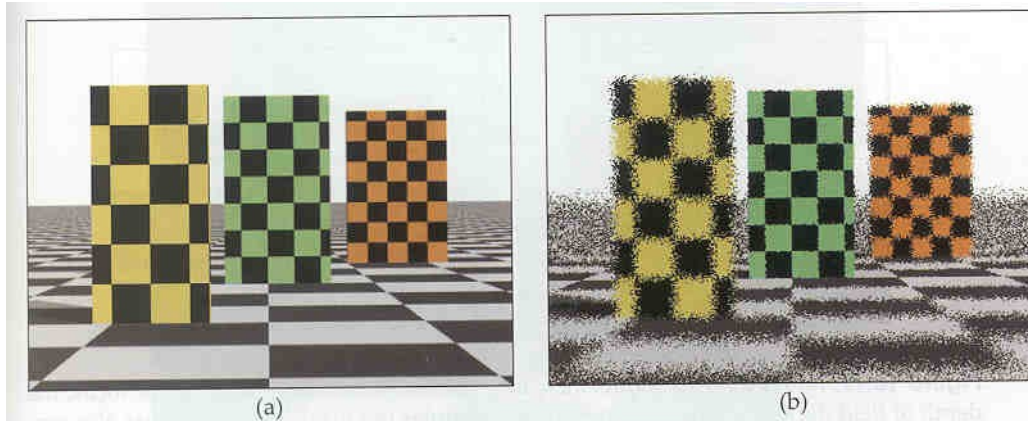
# Out of Focus Images



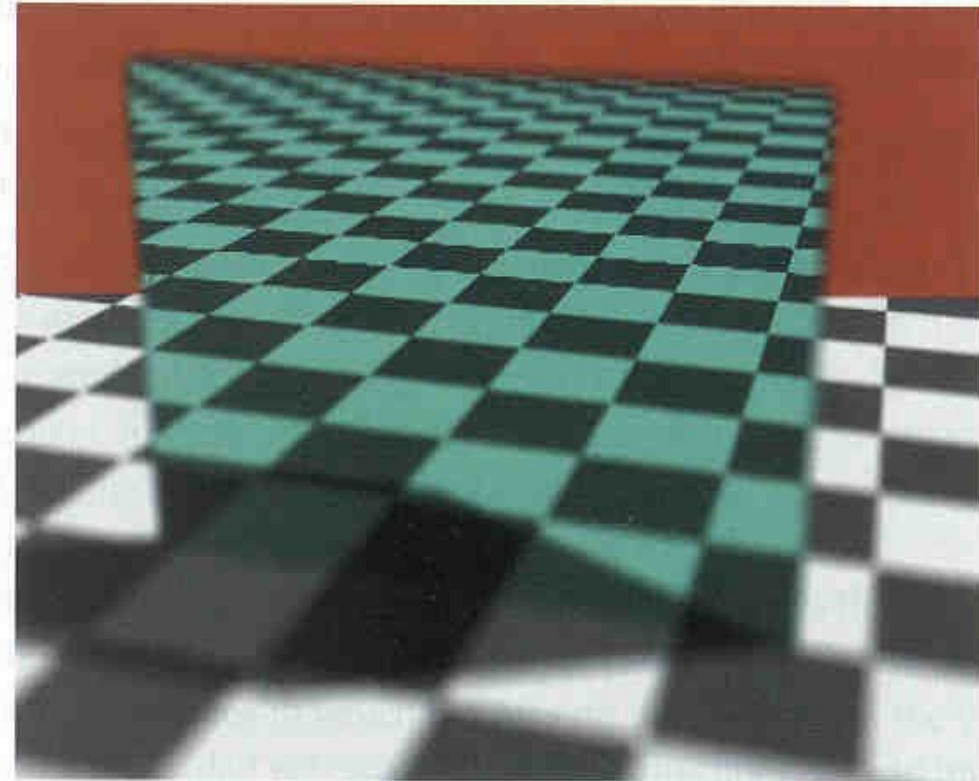
**Figure 10.2.** Rays starting a point  $q$  go through the image plane of  $p$  at different locations, with the result that  $q$  will appear out of focus.



# Depth of Field Results



**Figure 10.9.** (a) When the lens radius is zero, the image is the same as a pinhole-camera image with everything in focus; (b) noisy image from using one random sample per pixel.



**Figure 10.12.** Mirrored surface.

# Ambient Occlusion

- Floor has a vague shadow outline
- Parts of object near floor is darker
- Ambient light is not anisotropic and uniform

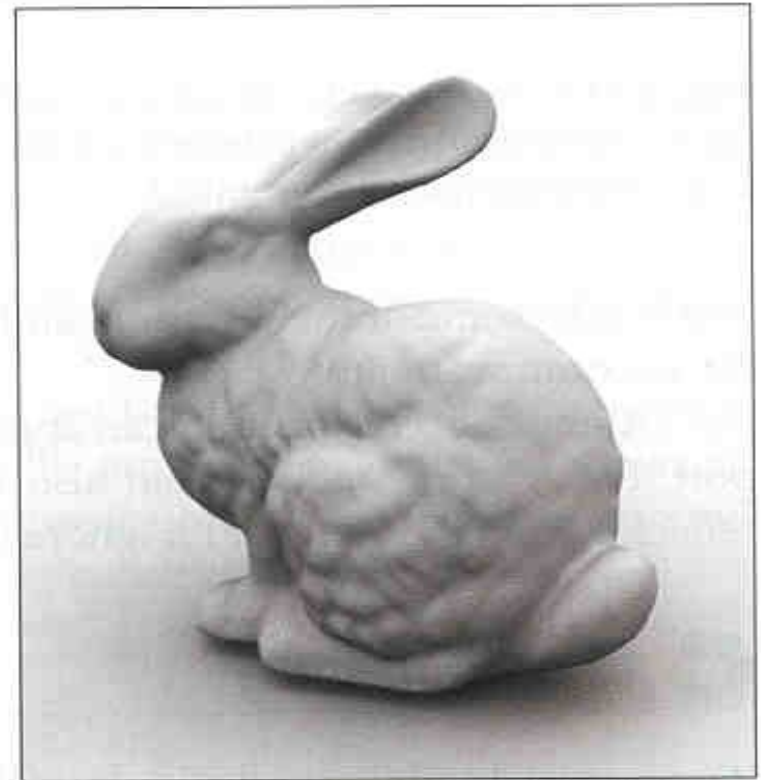
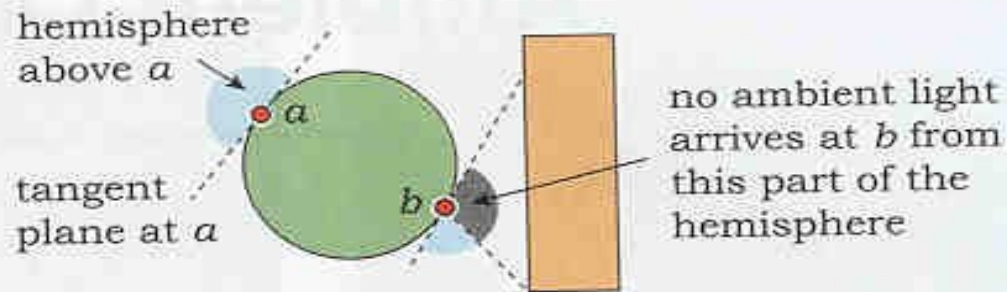
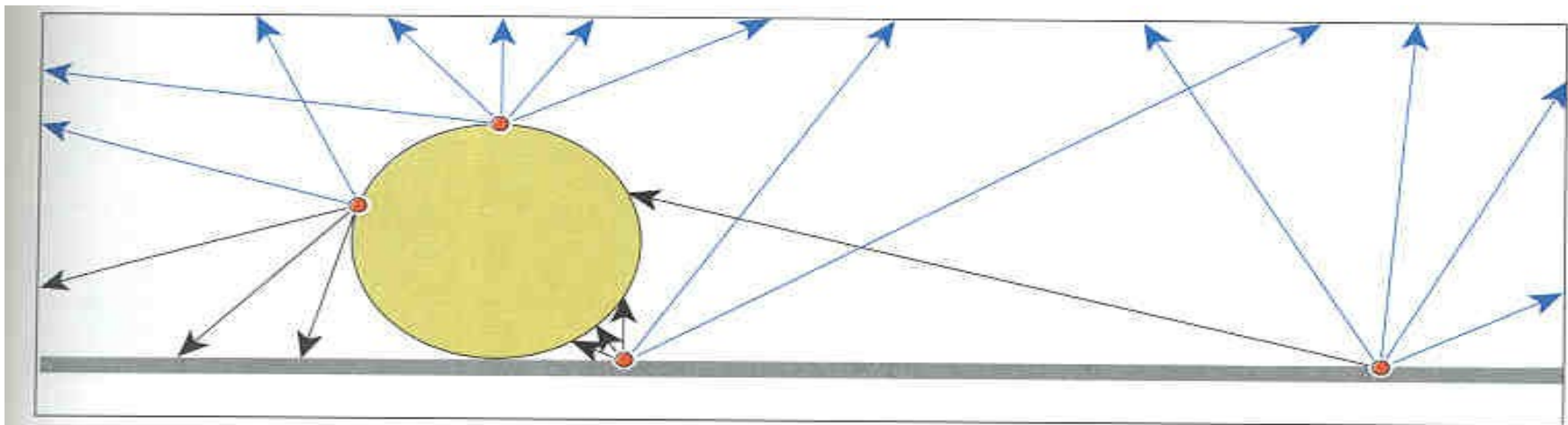


Image courtesy of Mark Howard, Stanford bunny  
model courtesy of Greg Turk and the Stanford  
University Graphics Laboratory

# Computing Ambient Occlusion

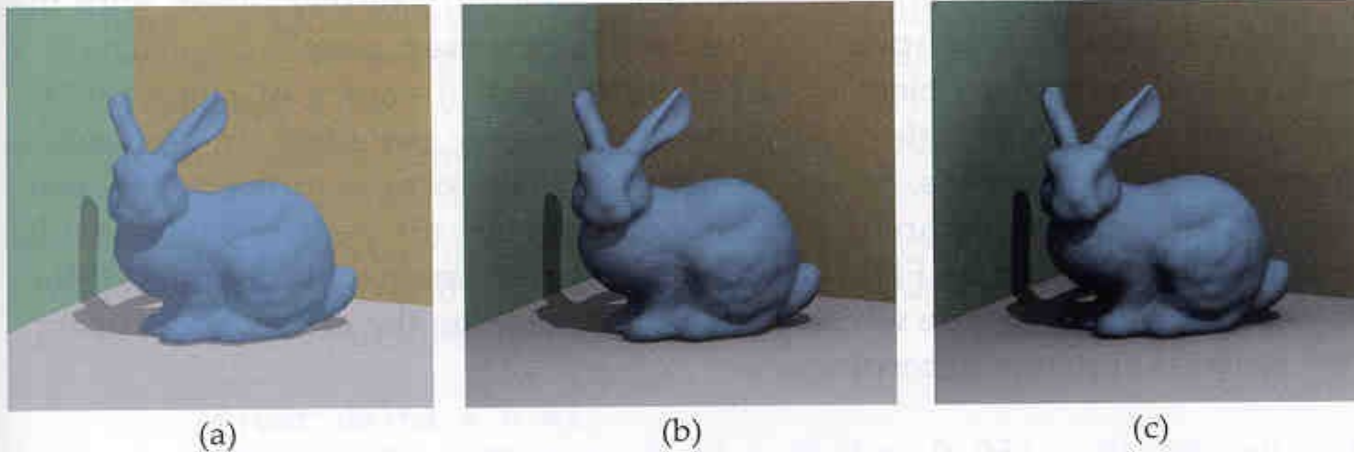


**Figure 17.1.** Point  $a$  on the sphere receives the maximum amount of ambient light because the box isn't visible; point  $b$  doesn't receive the maximum amount because the box blocks some of the incoming ambient light.

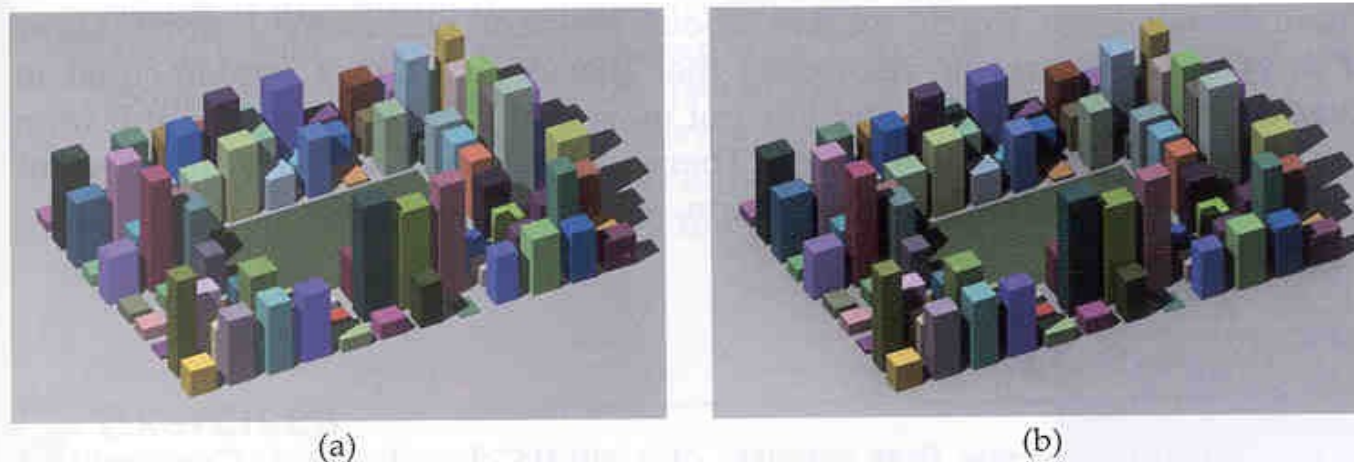


**Figure 17.2.** Various hit points on the plane and the sphere, with sample shadow rays.

# Ambient Occlusion Results



**Figure 17.12.** Bunny scene rendered with 256 samples per pixel: (a)  $\text{min\_amount} = 1$ ; (b)  $\text{min\_amount} = 0.25$ ; (c)  $\text{min\_amount} = 0$ .



**Figure 17.13.** Random boxes rendered with 64 samples per pixel: (a)  $\text{min\_amount} = 1.0$ ; (b)  $\text{min\_amount} = 0.25$ .

# Mirror Reflection

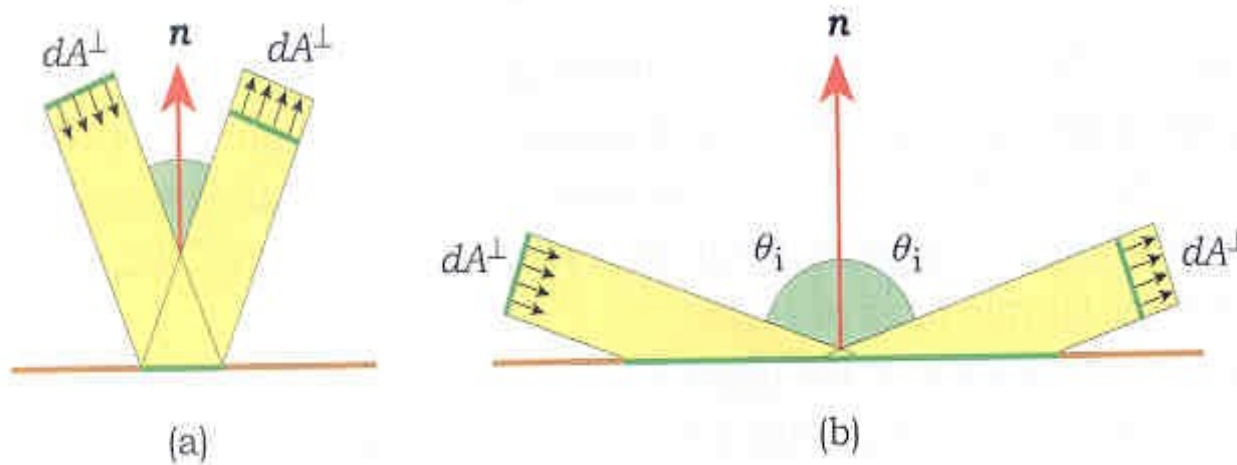
- Mirror reflections are a signature of ray tracing
  - Shiny objects
    - Glass
    - Metal
  - Multiple reflections may occur
- Occurs naturally in ray tracing
- Requires tracing ray through multiple bounces
- Adds significant effort



Reflective spheres by Burt Flugleman, Rundle Street Mall, Adelaide. Photograph by Kevin Suffern.

# Conservation of Energy

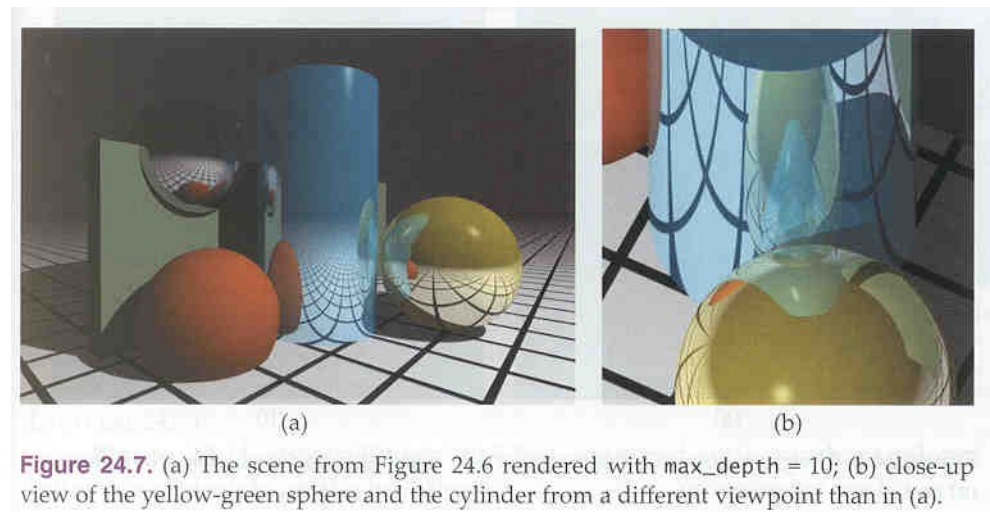
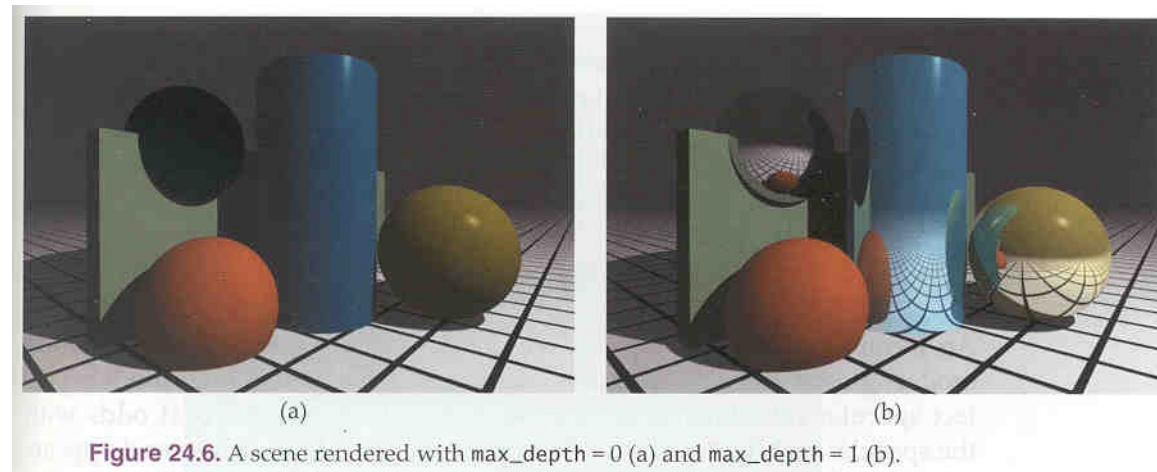
- Mirrors reflect almost all the energy
- Retains beam geometry



**Figure 24.3.** When a beam of light is reflected from a perfect mirror, its cross section area is unchanged after reflection and is therefore independent of the angle of incidence  $\theta_i$ .

# Number of Reflections

- 0 dull
- 1 “simple” mirror
- $>1$  “hall of mirrors”
- Effort grows with number of bounces



# Hall of Mirrors

(Showcases Ray Tracing)



(a)



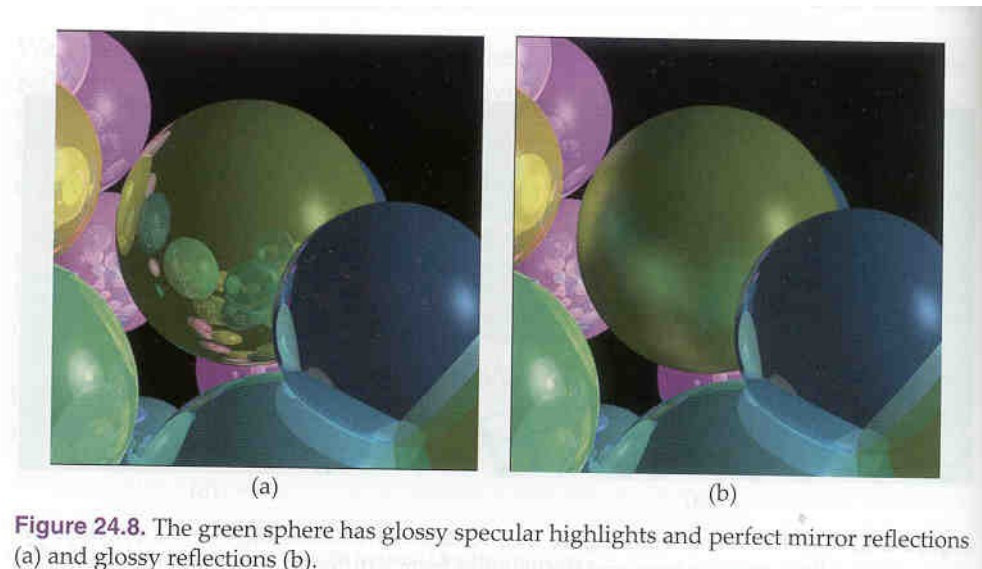
(b)

**Figure 24.18.** (a) Hall of mirrors with `max_depth = 19`; (b) close-up view of the multiple reflections between the floor mirror and the sphere.

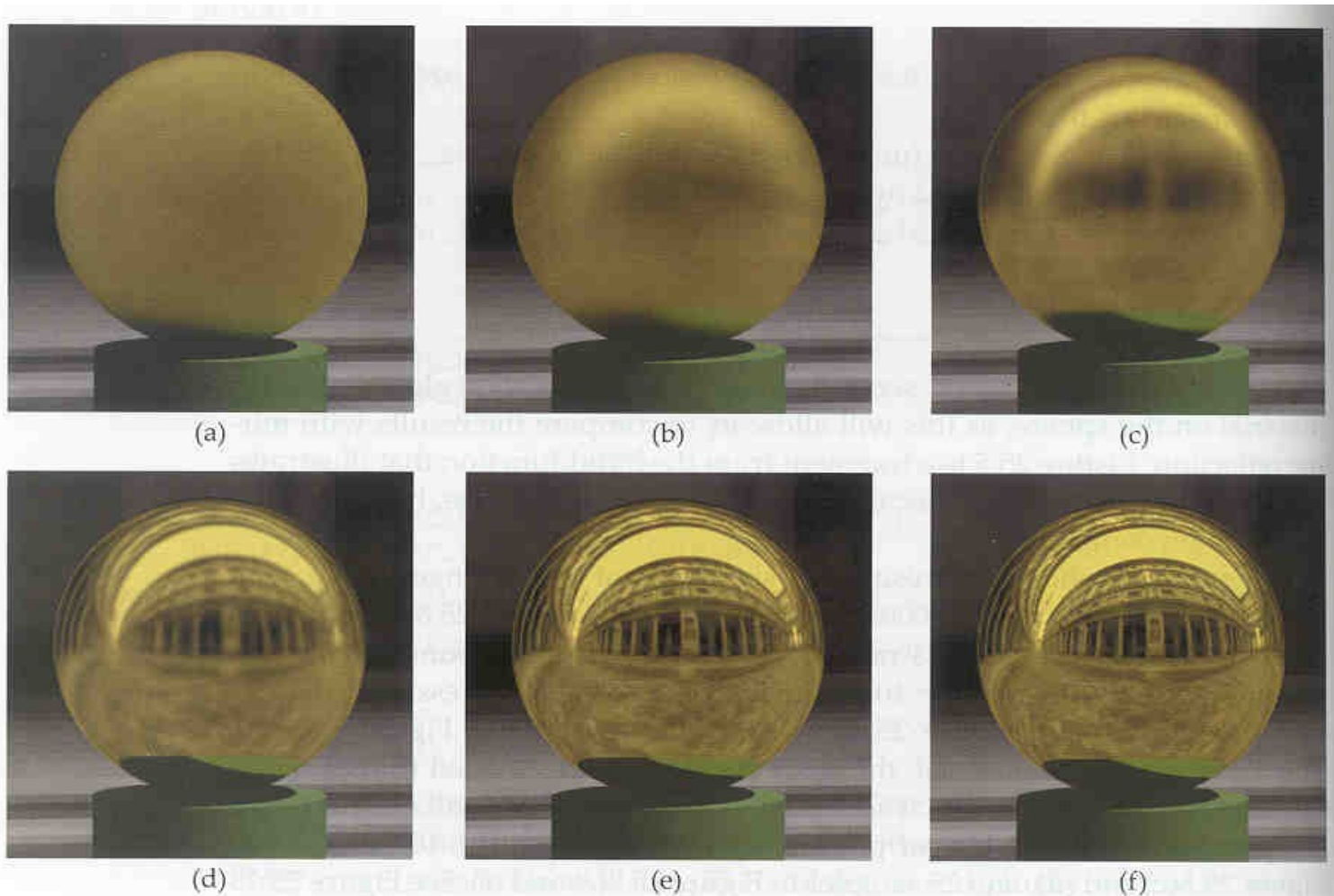


# Mirror vs Glossy Reflection

- Mirror reflections are “perfect”
- Glossy reflections are “imperfect”
  - Reflected ray =  $2(N \cdot V)N - V + \epsilon$
  - Super-sample for many values of  $\epsilon$



# Degrees of perfection



**Figure 25.8.** Glossy sphere surrounded by the Uffizi image and rendered with the following values of  $\epsilon$ : (a) 1.0; (b) 10.0; (c) 100.0; (d) 1000.0; (e) 10000.0; (f) 100000.0.

# Simple Transparency

- Light passes through objects
- Light changes through object
  - Rays are bent
  - Colors are changed
- Rays multiply
  - Reflected
  - Transmitted



Photograph courtesy of Steve Agland

# Refraction

- Index of refraction  $\eta = c/v$ 
  - Vacuum 1
  - Air 1.0003
  - Water 1.33
  - Glass 1.5
  - Diamond 2.42
- Snell's law
  - $\sin\theta_i / \sin\theta_t = \eta$

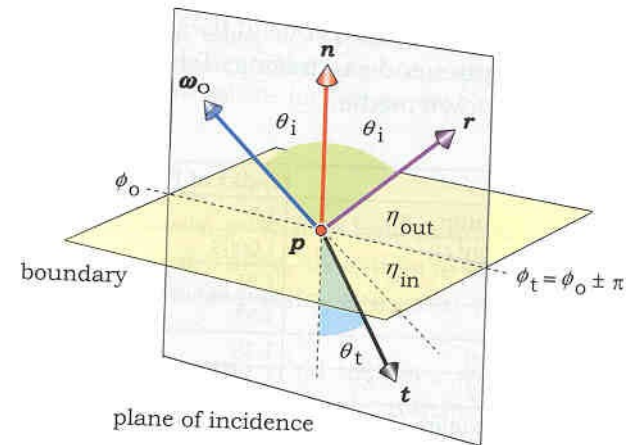


Figure 27.2. Reflected and transmitted rays at the boundary between two transparent media.

# Media Transitions

- Direction of bend depends on whether the refraction index increases or decreases
  - Air  $\eta$  is very low
  - Angles decrease into liquids
  - Angles increase out of liquids

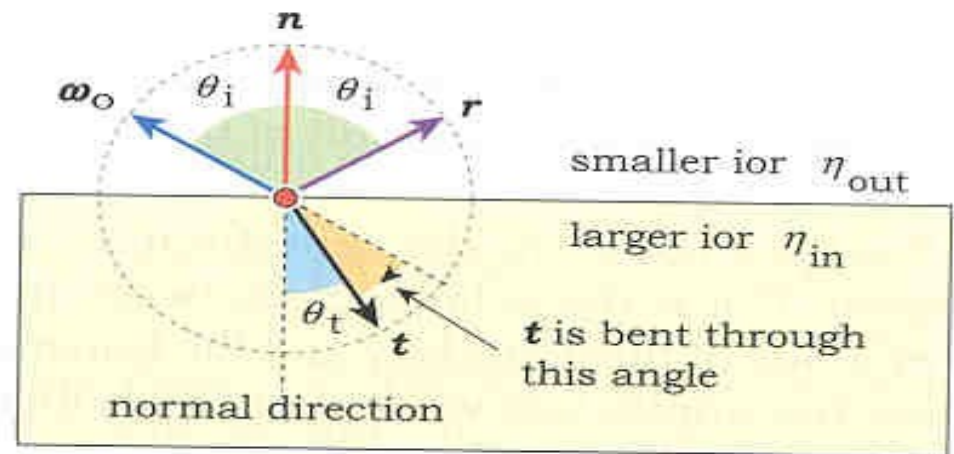


Figure 27.3. Direction change of  $t$  when  $\eta > 1$ .

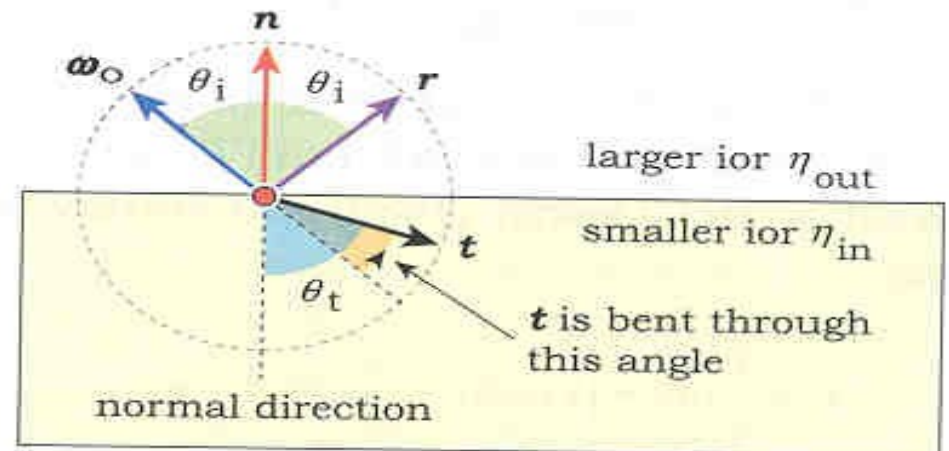


Figure 27.4. Direction change of  $t$  when  $\eta < 1$ .

# Internal reflections

- Critical angle
  - Refraction bends ray back into medium
- Higher  $\eta$  contrast causes larger critical angle
  - That is why diamonds are so sparkly

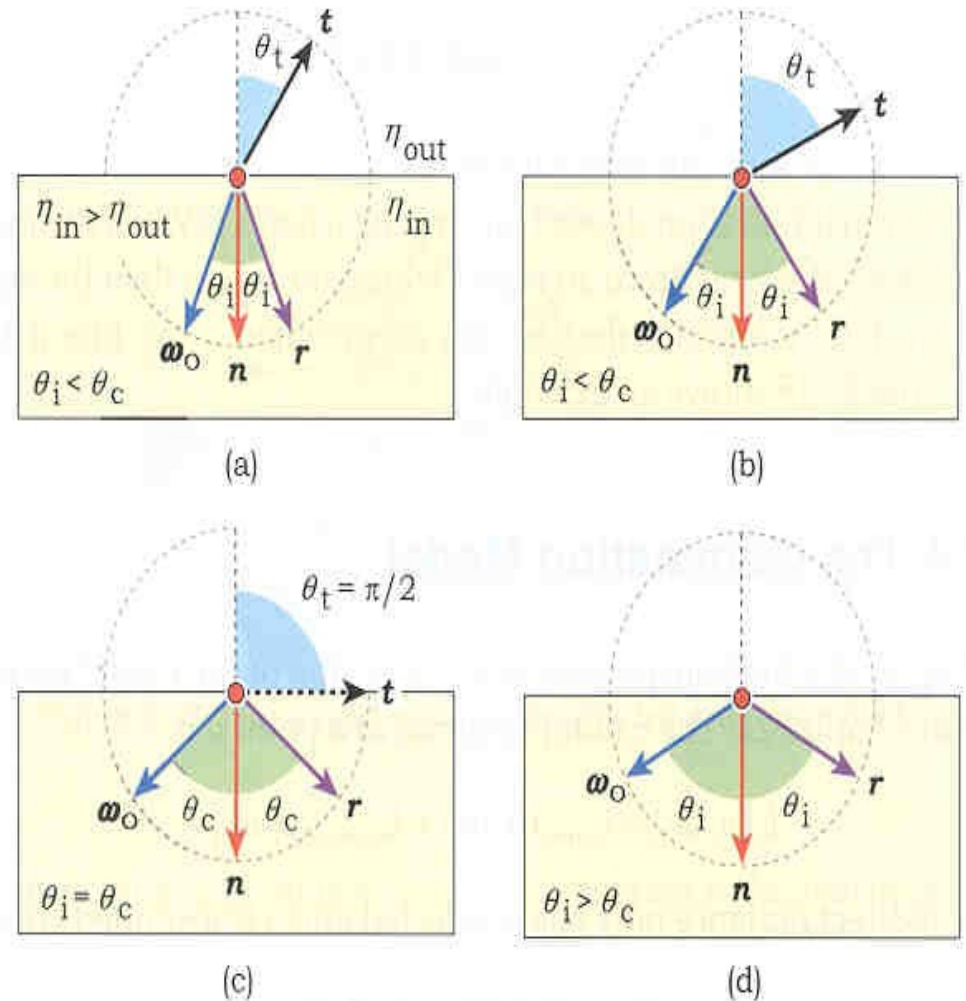


Figure 27.5. Total internal reflection: (a) and (b)  $\theta_i < \theta_c$ ; (c)  $\theta_i = \theta_c$ ; (d)  $\theta_i > \theta_c$ .

# Transparency require bifurcating rays

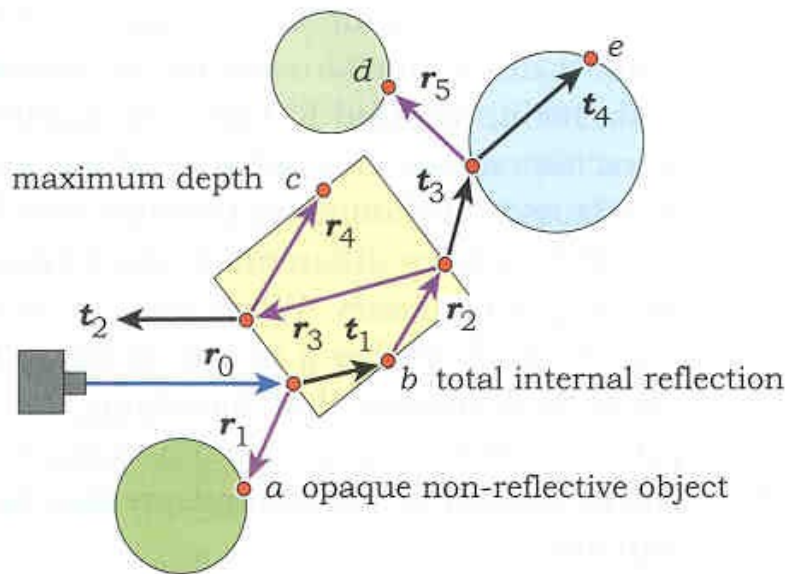


Figure 27.6. Transparent objects with reflected and transmitted rays.

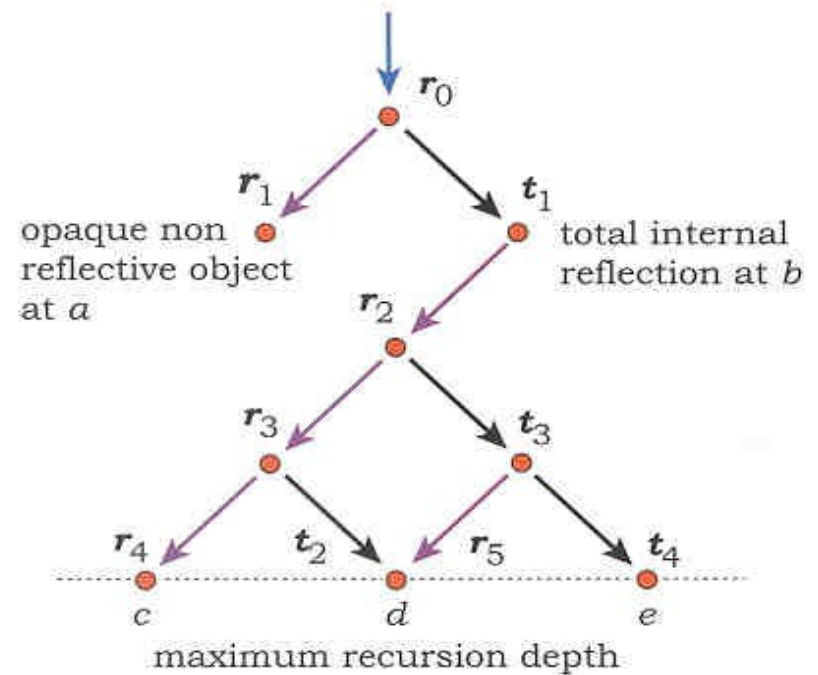
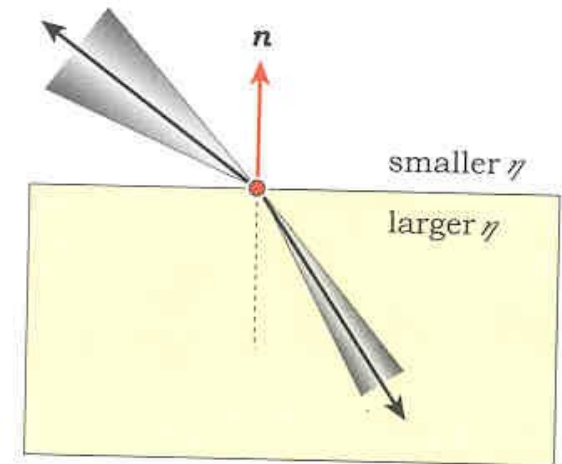


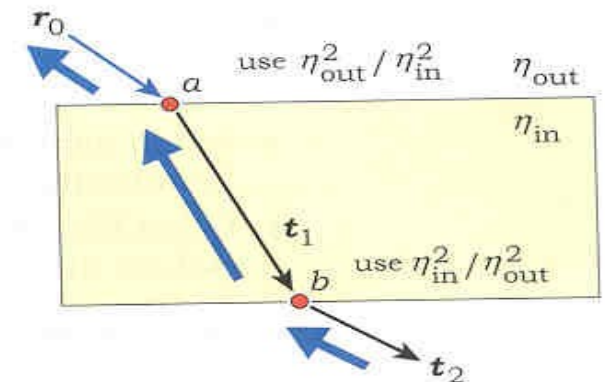
Figure 27.7. The ray tree that corresponds to Figure 27.6.

# Objects Appearance

- Object inside other material
  - Objects are magnified when not viewed parallel to the normal
  - Object's apparent position is displaced
- Objects on other side
  - Objects apparent position is displaced



**Figure 27.8.** The angle of a differential cone of incident radiance changes as it crosses the boundary between two dielectrics.



**Figure 27.9.** Ray and radiance-transfer directions through a transparent object.



# Distortion by Glass Spheres

- Sphere as a lens



Figure 27.22. Transparent sphere in front of text.

- Eye position is critical

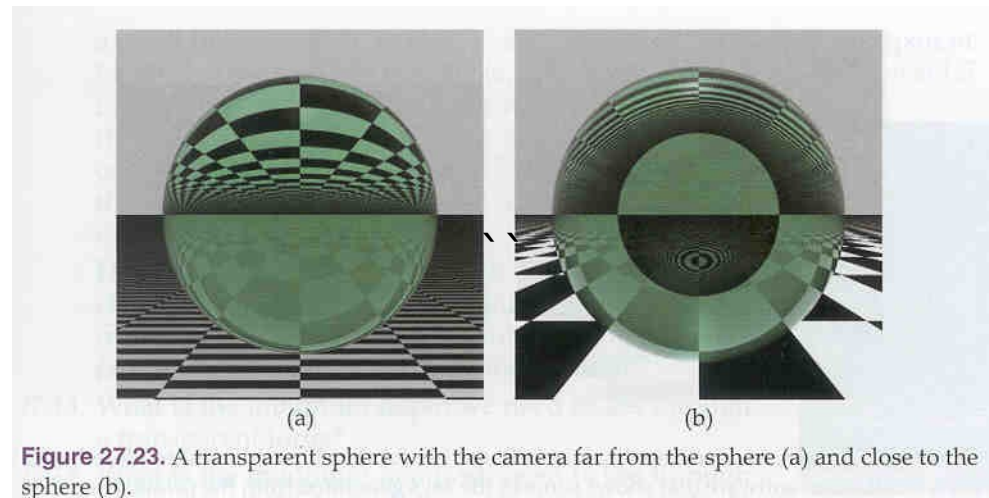


Figure 27.23. A transparent sphere with the camera far from the sphere (a) and close to the sphere (b).

# Light movement through sphere

- Magnification

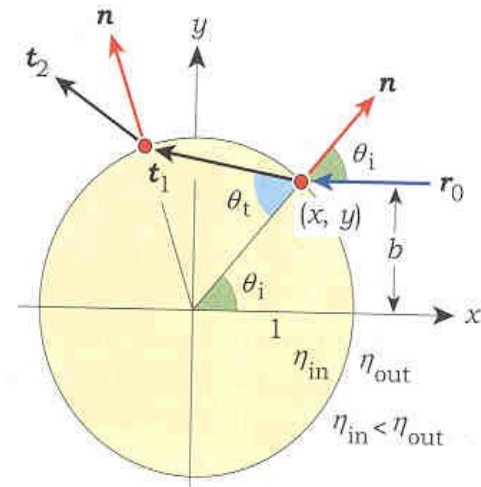


Figure 27.17. Reflected and transmitted rays generated by a ray  $r_0$  that hits a unit sphere with impact parameter  $b$ , where the sphere has  $\eta < 1$ .

- Internal reflection

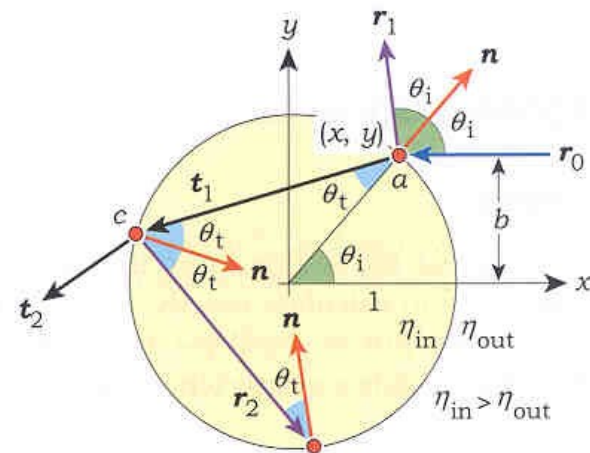


Figure 27.11. Reflected and transmitted rays generated by a ray  $r_0$  that hits a unit sphere with impact parameter  $b$ . The lengths of the (unit) normals and the sphere are not drawn on the same scale.

# Realistic Transparency

- Three  $\eta$ 's
  - Air
  - Glass
  - Water
- Colored liquid
- Beveled edges
  - Glass
  - Meniscus
- Mixed transparency
  - Foam is opaque

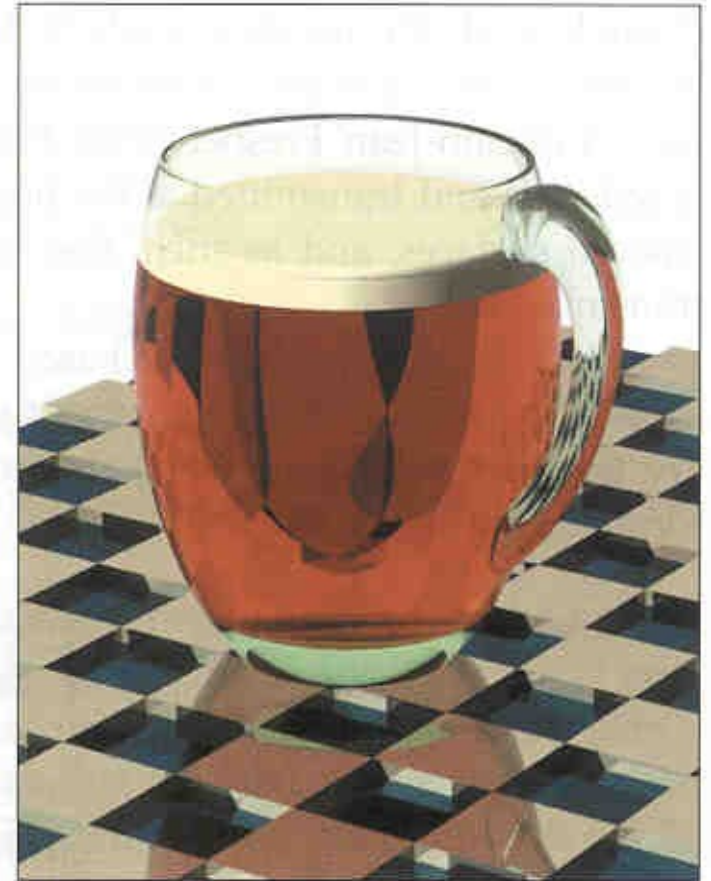


Image courtesy of John Avery

# Reflectance and Attenuation

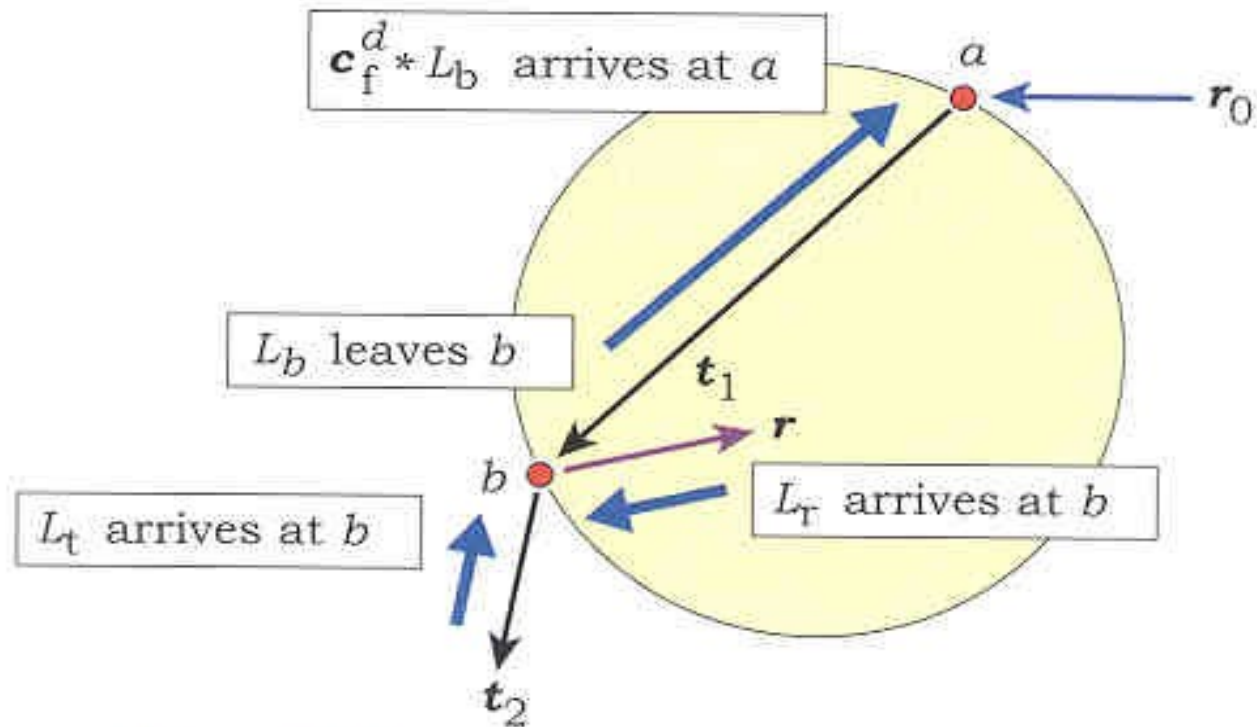


Figure 28.4. Radiance attenuation in a dielectric.

# Multiple Internal Reflections

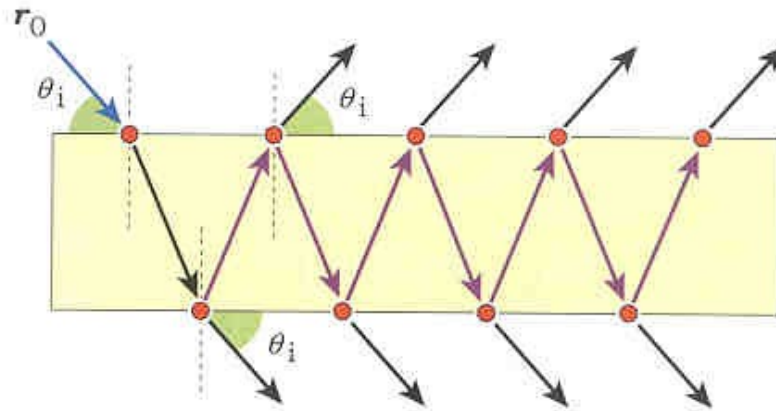


Figure 28.19. A transparent box with multiple reflected and transmitted rays.

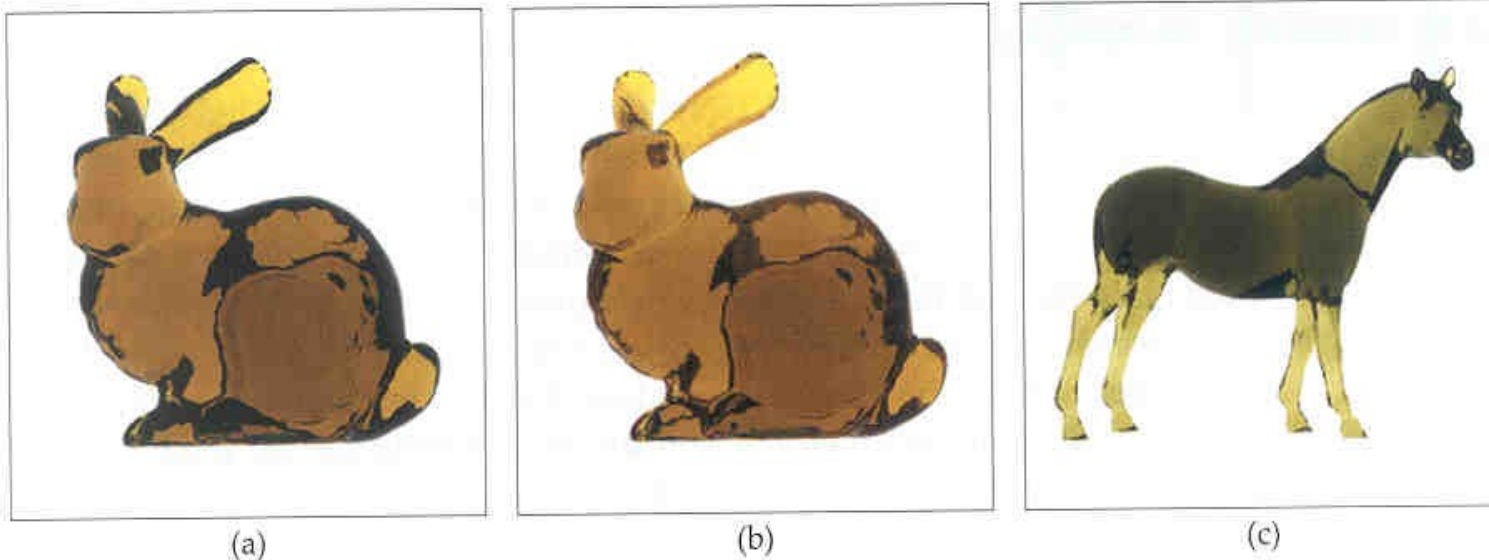
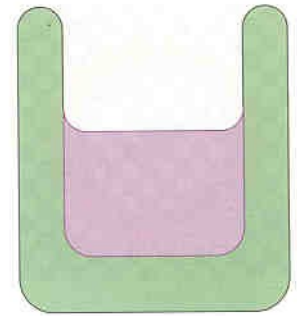


Figure 28.12. (a) Stanford bunny rendered with  $c_r = (0.65, 0.45, 0)$  and  $\text{max\_depth} = 2$ ; (b)  $\text{max\_depth} = 10$ ; (c) horse model rendered with  $c_r = (0.65, 0.65, 0.1)$  and  $\text{max\_depth} = 10$ .

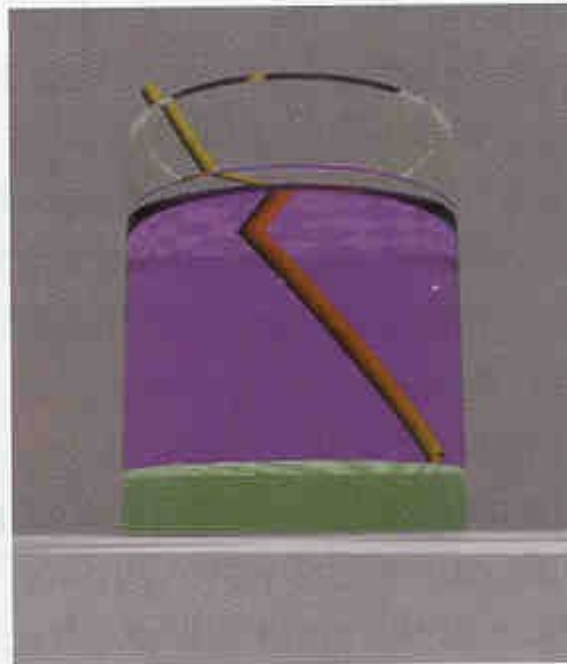
# Colored Beaker



**Figure 28.37.** A more sophisticated glass of water has a curved top, rounded edges, and a meniscus for the water.



(a)



(b)



(c)

**Figure 28.38.** Glass of water and straw rendered with: (a) no shadows; (b) camera looking up; (c) shadows and direct illumination on the straw.

# The Fish Bowl

- Making it real
  - Complex shape
  - Three media
  - Colored media
  - Beveled edges
- Challenges
  - Multiple reflections
  - Refraction

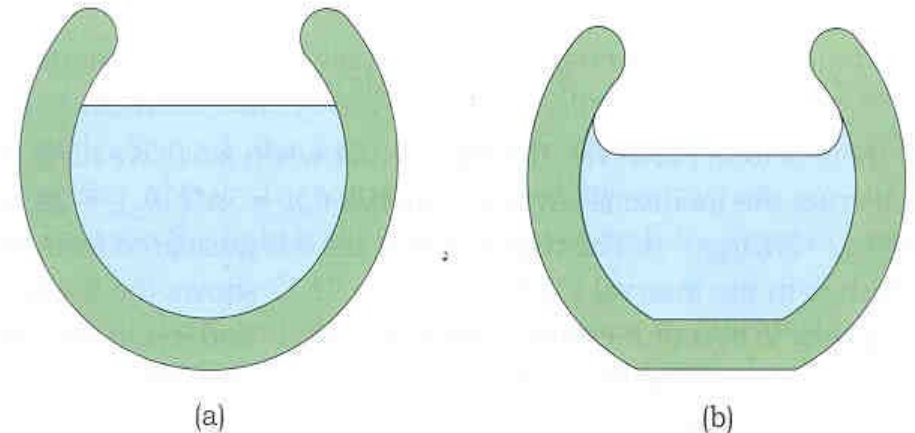


Figure 28.39. (a) Basic fishbowl; (b) fishbowl with flat base and meniscus.

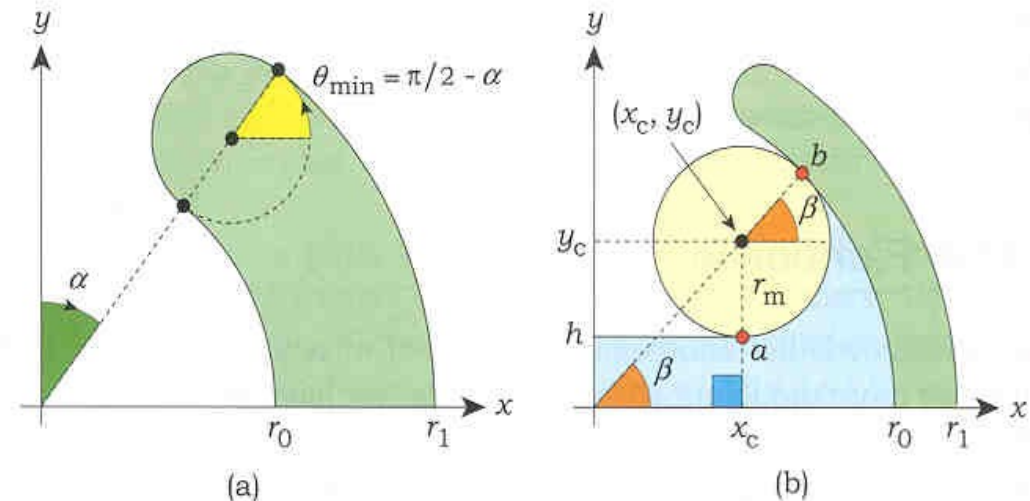
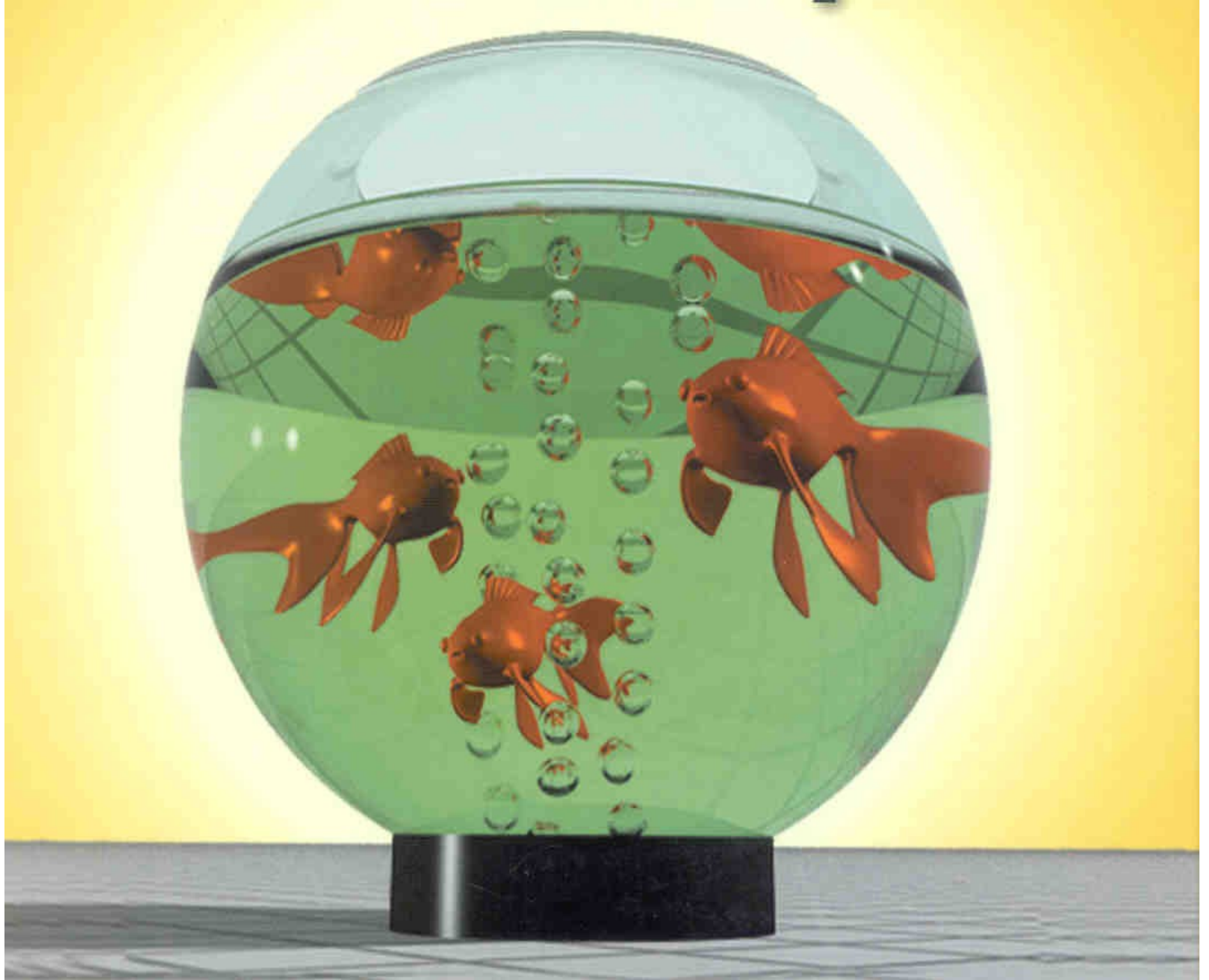


Figure 28.40. (a) Construction of the rim; (b) construction of the meniscus.





# Adding Textures

- Per pixel modification of surface appearance
- Use texture coordinates to map textures to objects
  - When ray tracing, you have to do this yourself
- Textures modify ray color on each bounce

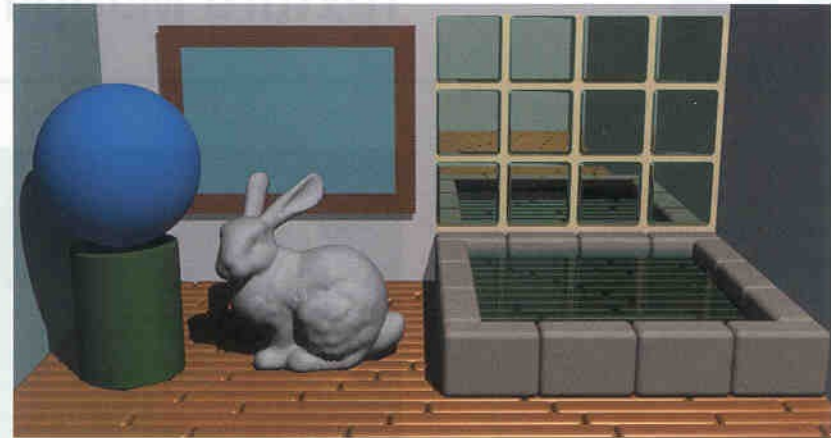
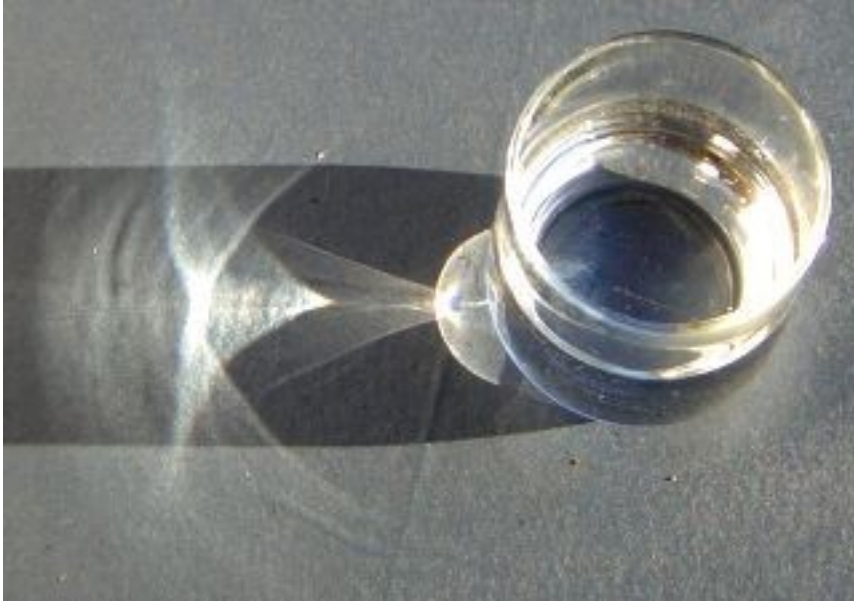


Figure 29.1. Interior scene rendered with no textures.

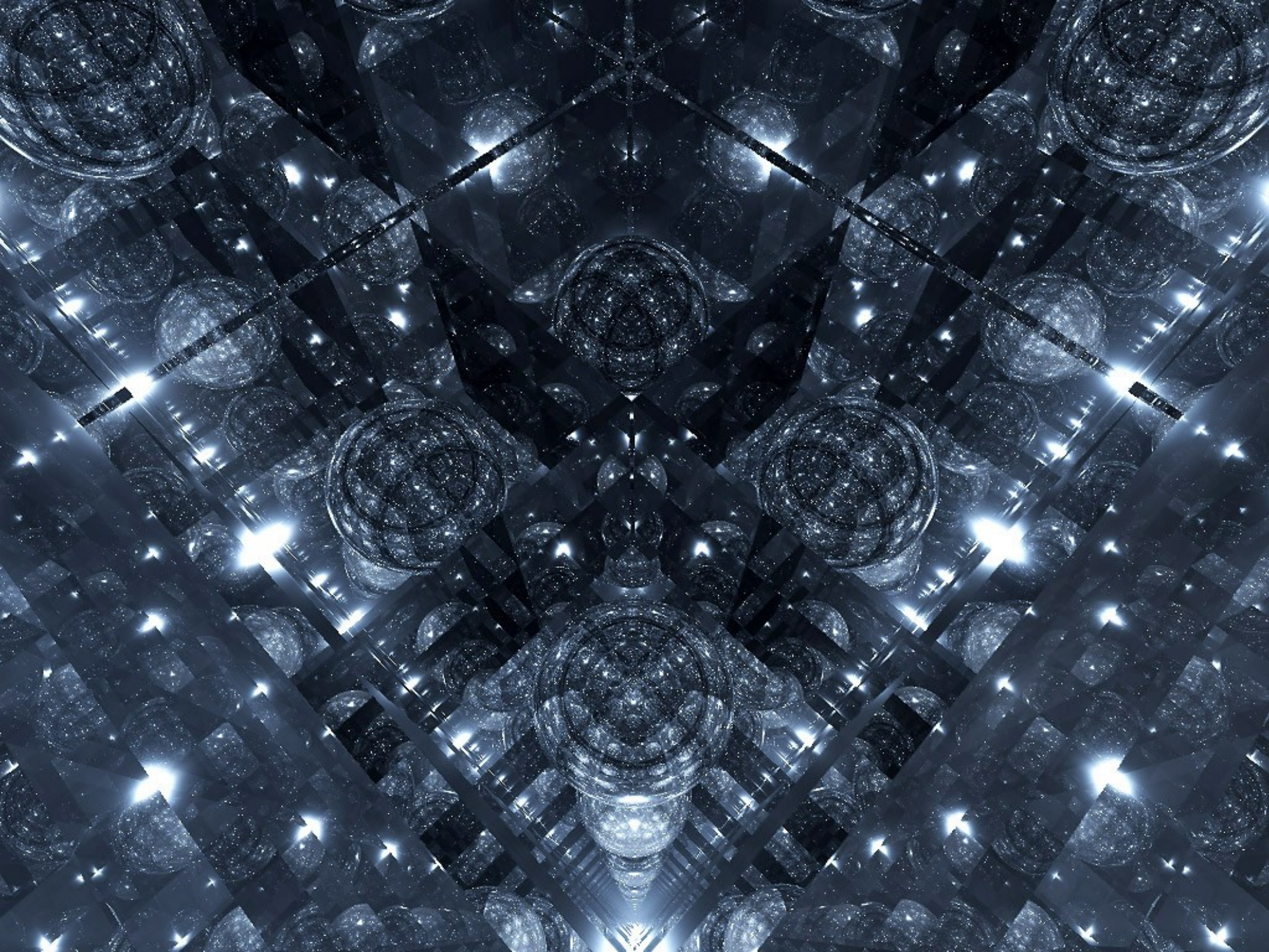


Figure 29.2. Same scene as in Figure 29.1 but rendered with a variety of textures. The water surface is Ken Musgrave's water bump map, as described in Musgrave (2003b).

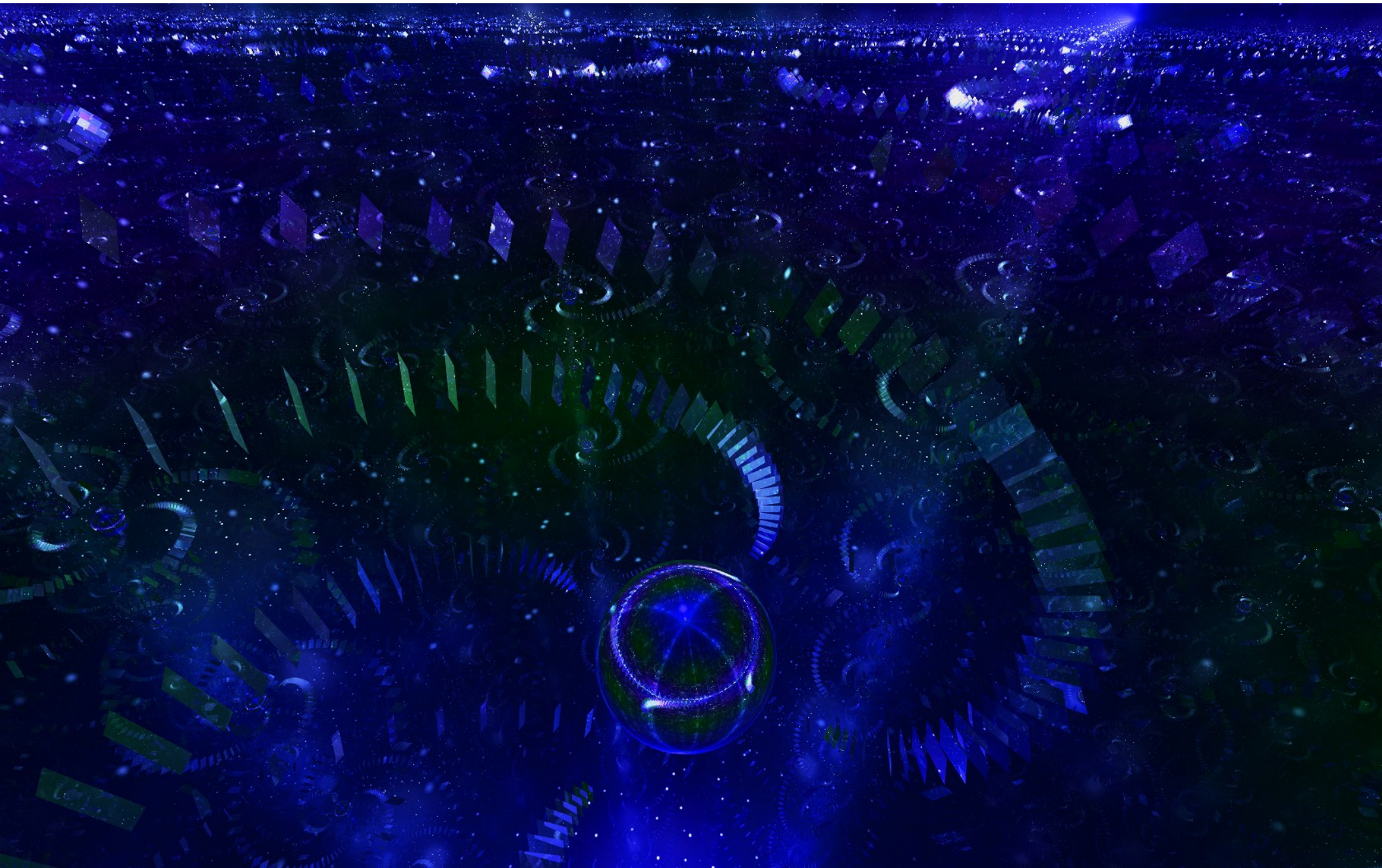
# Caustics

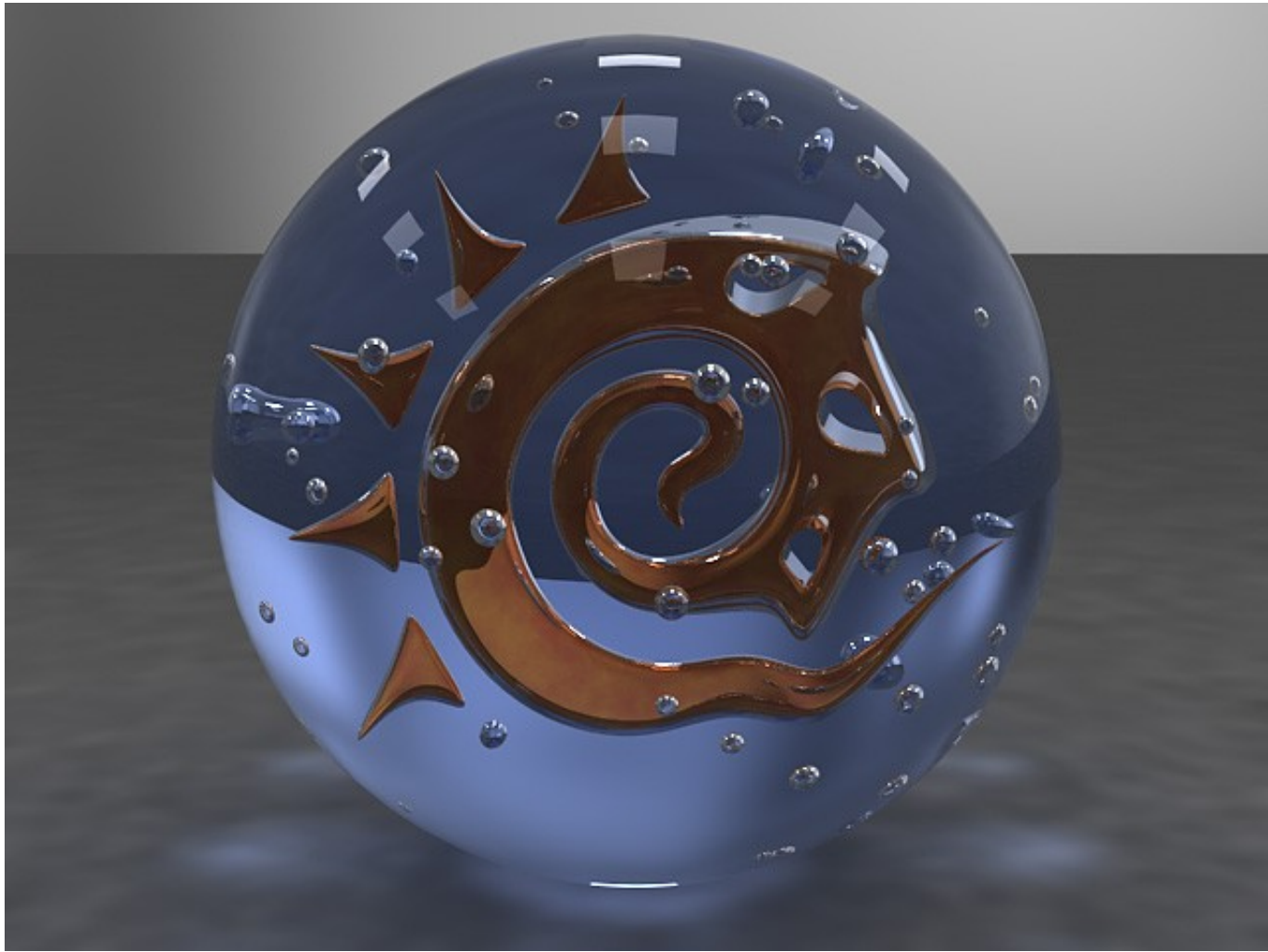


# Tim Dunn's Gallery











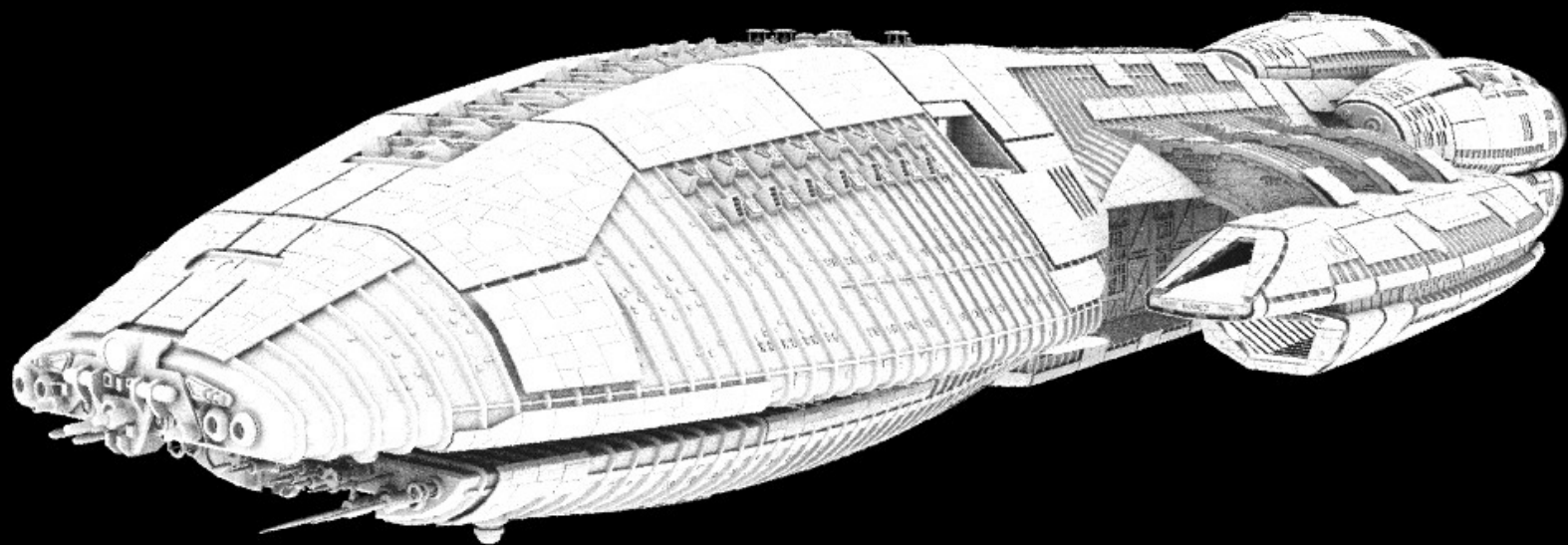


# Production Ray Tracing

## Tim Dunn

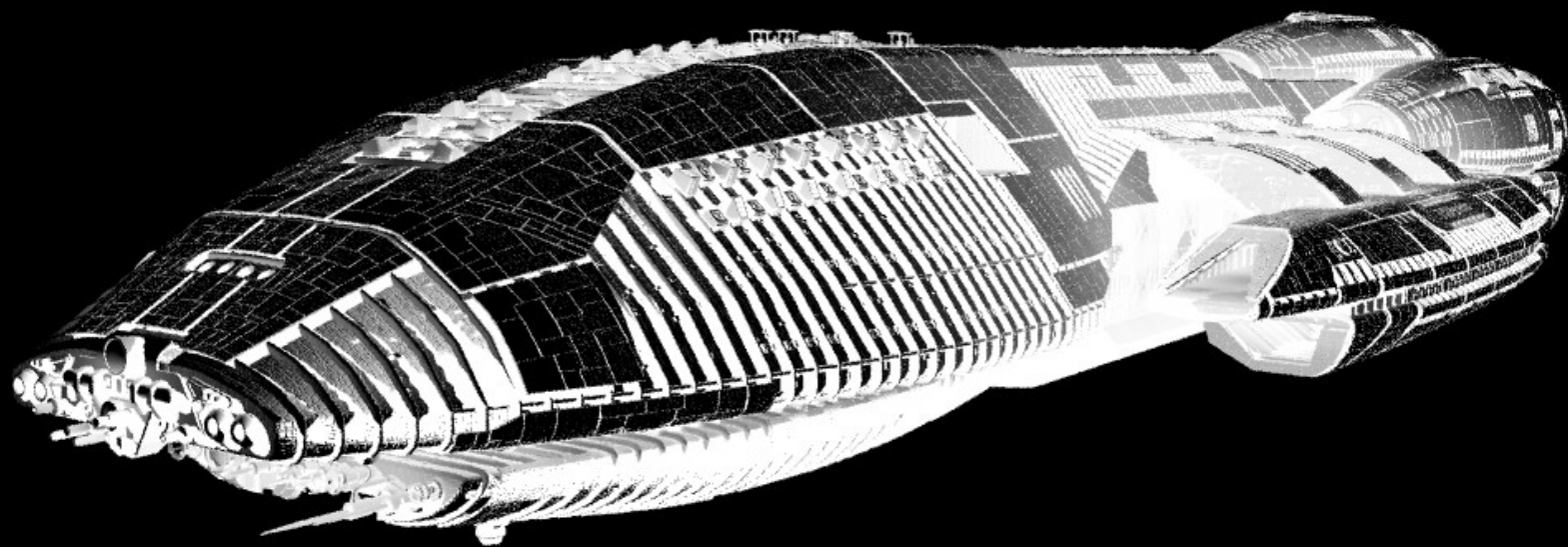


**Raw Render Pass**

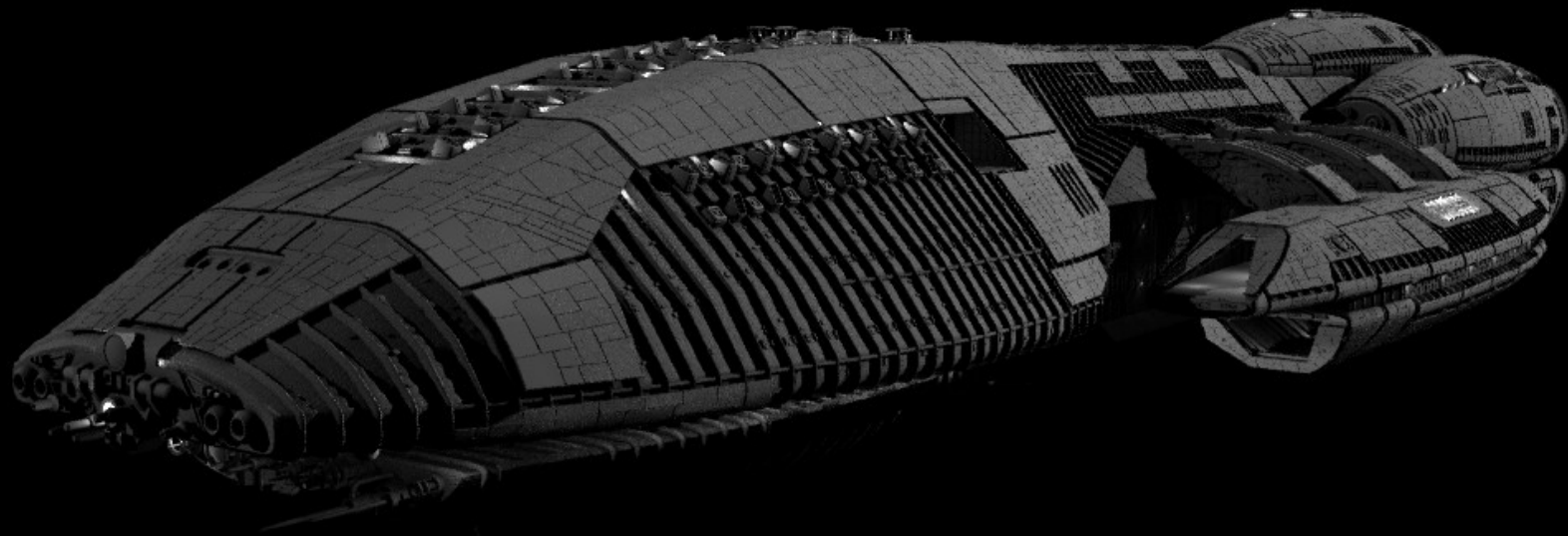


**Ambient Occlusion Pass**

**Luminosity Pass**



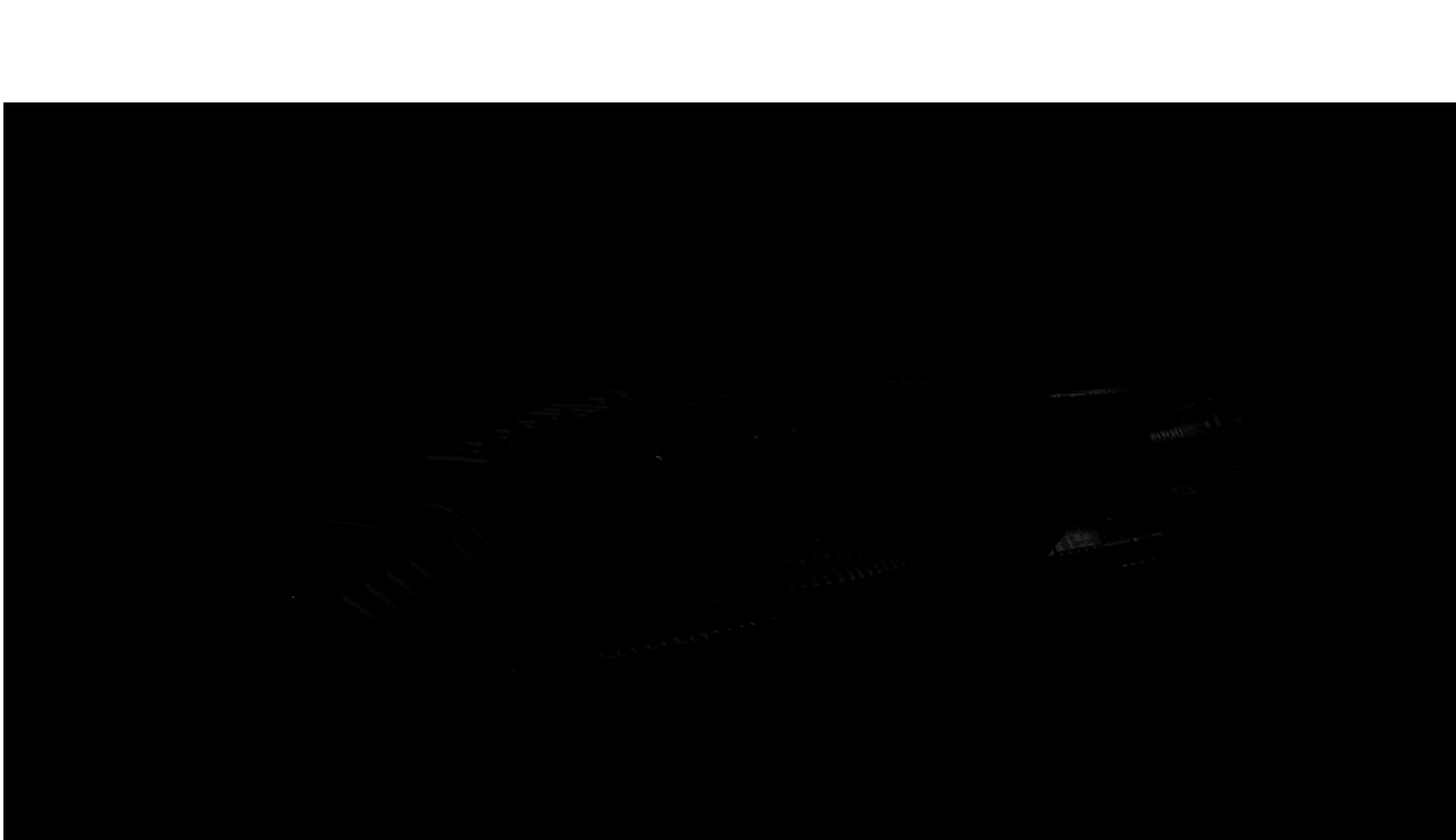
**Shadow Pass**



**Diffuse Pass**



**Dissue Lighting Pass**



**Specular Color Pass**





**Background Plate Pass**



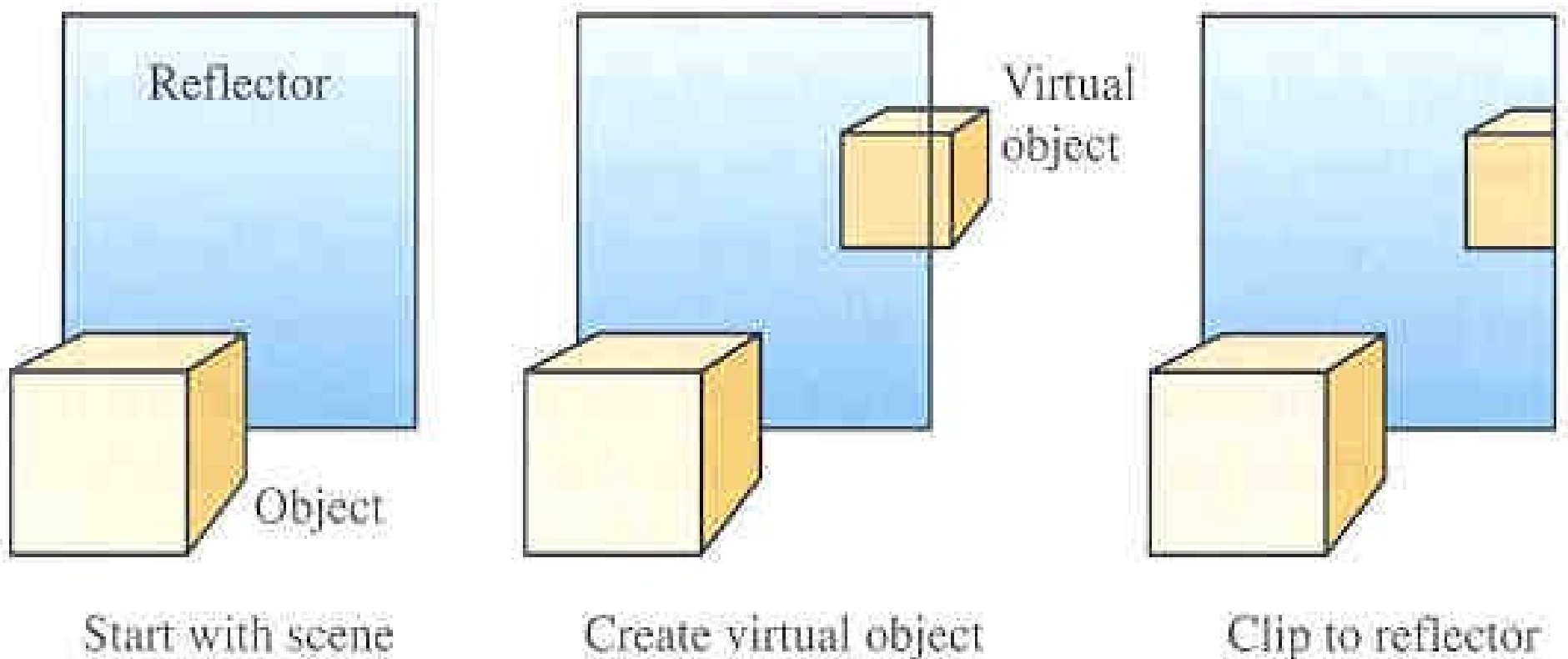
**Final Composite**

# Reflections in Raster Methods

- Two possible approaches
  - Textures (image space)
  - Virtual objects (object space)
- Both approaches requires rendering the scene multiple times
- Mirrors can be planar or curved
- Mirrors are “windows” to the virtual scene

# Virtual Objects

- Draw object where they seem to appear
- Clip to reflector



# Planar Reflection Equation

- Point on mirror  $P$
- Normal vector  $V$

$$R = \begin{pmatrix} 1 - 2V_x^2 & -2V_xV_y & -2V_xV_z & 2(P \cdot V)V_x \\ -2V_xV_y & 1 - 2V_y^2 & -2V_yV_z & 2(P \cdot V)V_y \\ -2V_xV_z & -2V_yV_z & 1 - 2V_z^2 & 2(P \cdot V)V_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Rendering Order

- Reflections are difficult when the mirror is an object inside the scene
  - Mirror on wall is easier

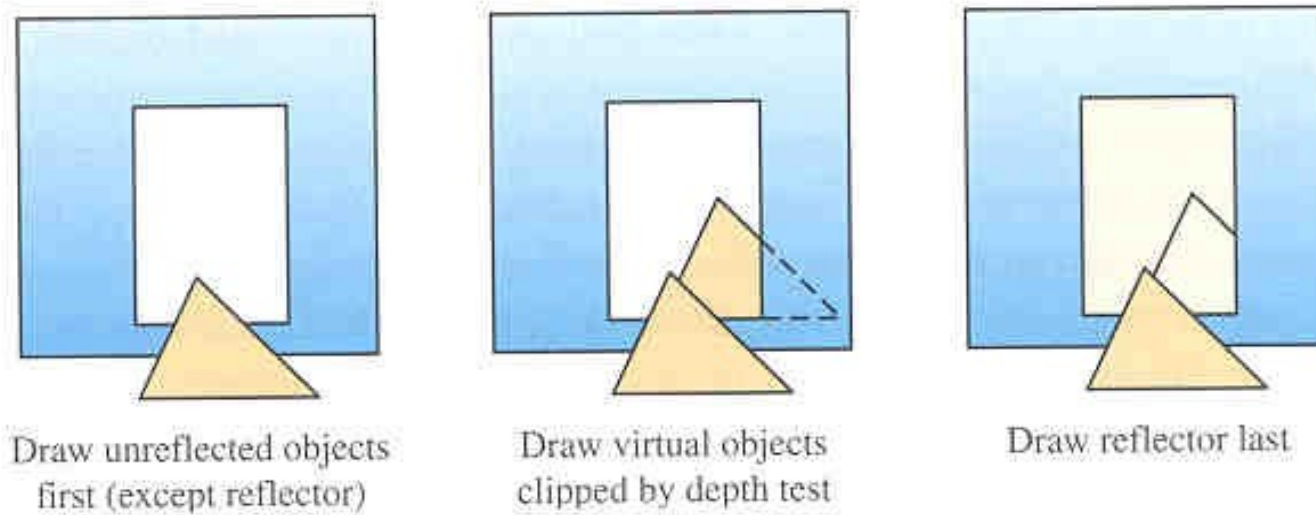


Figure 17.3 Masking reflections with depth buffering.

# Limiting the Reflector

- User defined clipping volume
  - Front and back clipping planes
  - Frustum
- Stencil buffer
- Special cases
  - Scissors test
  - Alpha blending

# Reflections using Textures

- Quads
  - Simple mirrors
- Environment maps
  - Cube map
  - Sphere map



Figure 17.4 Masking reflections using projective texture.



# Reflections from Curved Surfaces

- Cannot be done using virtual objects
- Readily done by distorting textures

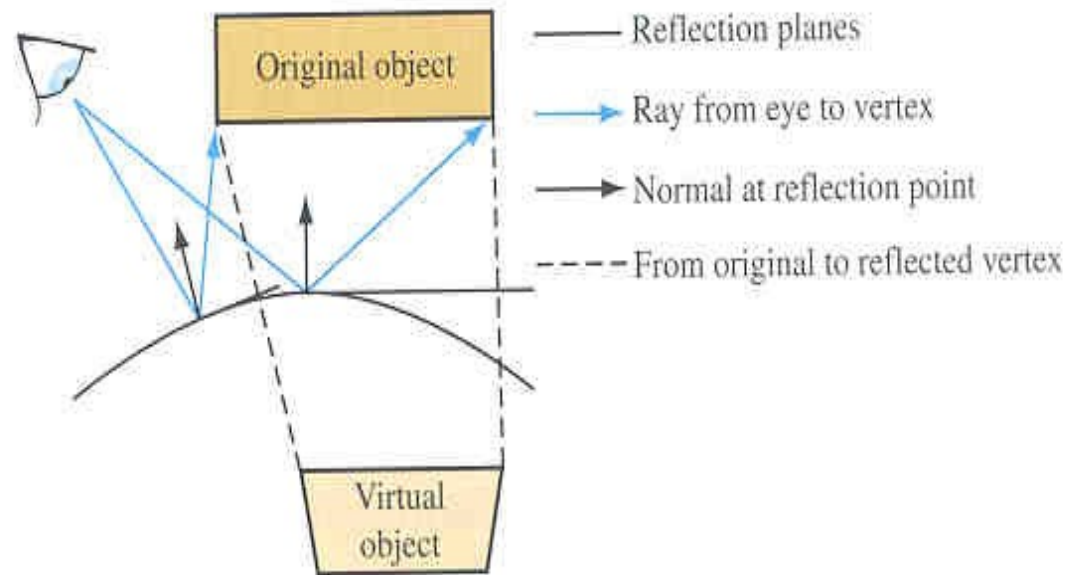


Figure 17.5 Normals and reflection vectors in curved reflectors.

# Inter-reflections

- Hall of mirrors requires multiple passes
  - Similar to max-levels

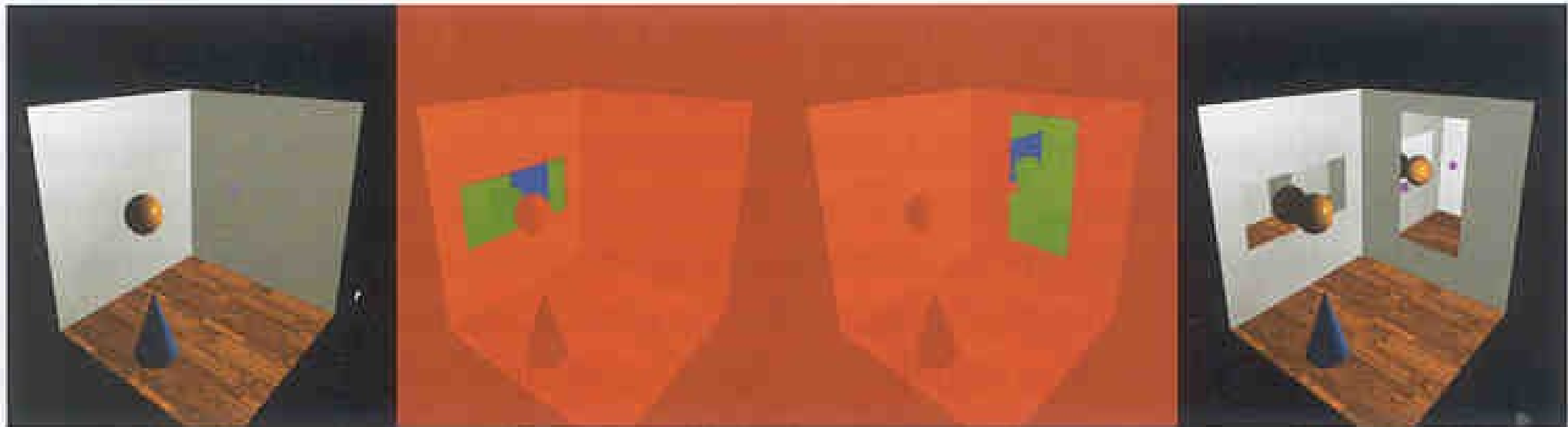


Figure 17.13 Clipping multiple interreflections with stencil.