Homogeneous Coordinates

CSCI 4229/5229
Computer Graphics
Summer 2009

2D Homogeneous Transformations

Translation:

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & \delta x \\ 0 & 1 & \delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Scaling:

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Rotation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

3D Homogeneous Translation and Scaling

Translation:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Scaling:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

3D Homogeneous Rotation

Rotation around the z axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotation around the x axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotation around the y axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

General Homogeneous Rotation

Rotation around a unit vector (X, Y, Z):

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 + (1 - \cos\theta)(X^2 - 1) & -Z\sin\theta + (1-\cos\theta)XY & Y\sin\theta + (1-\cos\theta)XZ & 0 \\ Z\sin\theta + (1-\cos\theta)XY & 1 + (1-\cos\theta)(Y^2 - 1) & -X\sin\theta + (1-\cos\theta)YZ & 0 \\ -Y\sin\theta + (1-\cos\theta)XZ & X\sin\theta + (1-\cos\theta)YZ & 1 + (1-\cos\theta)(Z^2 - 1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Rotate x axis to unit vector (X_0, Y_0, Z_0) and y axis to unit vector (X_1, Y_1, Z_1):

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} X_0 & X_1 & Y_0Z_1 - Y_1Z_0 & 0 \\ Y_0 & Y_1 & Z_0X_1 - Z_1X_0 & 0 \\ Z_0 & Z_1 & X_0Y_1 - X_1Y_0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

3D Homogeneous Transformation

Transformation matrices will always be of the form

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} r_{xx} & r_{yx} & r_{zx} & t_x \\ r_{xy} & r_{yy} & r_{zy} & t_y \\ r_{xz} & r_{yz} & r_{zz} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Order of calls in OpenGL

- What is the resulting $x' = M_1 M_2 M_3 x$ for
  - glTranslate(1000,2000,3000);
  - glRotate(90,0,0,1);
  - glScale(10,20,30);
  - .....
  - glVertex(1,0,0);
- Answer: $x' = (1000,2010,3000)$
- Killer fact: OpenGL RIGHT multiplies, so read from glVertex **backwards**

OpenGL Transformation Matrices

- You can set the matrices explicitly in OpenGL
  - glLoadMatrixd(double mat[16])
  - glMultMatrixd(double mat[16])
- Setting the matrix may be convenient when you calculate vectors instead of angles
- Killer fact:
  - The matrix is stored in COLUMN major order

Useful Vector Results

- **c** = **a** x **b**
  - **c** is perpendicular to the plane formed by **a** and **b**
  - **c** is zero when **a** and **b** are parallel
- **c** = **a** x (**b** x **a**)
  - **c** is perpendicular to **a**
  - **c** is in the plane formed by **a** and **b**
  - **c** is zero when **a** and **b** are parallel